A STUDY OF GAS FLOW IN THE "COMPREX" BY A HYDRAULIC ANALOGY

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Professor J.S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the Degree of Master of Science in Naval Construction and Engineering, we submit herewith a thesis entitled, "A Study of Gas Flow in the 'Comprex' by a Hydraulic Analogy."

Respectfully,

ACKNOWLEDGMENT

The authors wish to express their apprecia-tion to Professor A. H. Shapiro for his valuable advice and assistance, and for the suggestion which originally inspired this investigation. The authors wish also to express their apprecia-tion to the personnel of the Boston Naval Ship-yard without whose cooperation and assistance the experimental equipment could not have been completed.

TABLE OF CONTENTS

NOMENCLATURE

Symbol	Quantity Represented
A	Cross sectional area of water stream
$H=\frac{V^2}{2g}+h$	Total head of liquid
M	Mach Number
Q	Weight rate of flow of water
T	Absolute temperature
V	Velocity
W	Weight
a	Velocity of sound in gas
b	Width of water stream at free surface
c	Velocity of propagation of small disturbance in water
g	Gravitational acceleration
h	Height of water level above bottom of channel
$k=\frac{c_p}{c_v}$	Ratio of specific heats of gas
p	Pressure
t	Time
x,y	Cartesian coordinates
z	Distance from free surface to center of gravity of cross section of water stream
ρ	Density

Subscripts

a	Gas
w	Compression wave

x		Conditions at high water level, observer stationary with respect to channel
0	(1)	Conditions at low water level, observer stationary with respect to channel
	(2)	Stagnation conditions for gas
1	(1)	Conditions at low level, observer stationary with respect to first compression wave
	(2)	Conditions in gas stream approaching shock
2	(1)	Conditions at intermediate level, observer stationary with respect to first compression
	(2)	Conditions in gas stream leaving shock
22		Conditions at intermediate level, observer stationary with respect to second compression wave
33		Conditions at high level, observer stationary with respect to second compression wave

I. SUMMARY

The purpose of this thesis is to study gas flow in the "Comprex" by application of the analogy between gas flow in pipes and water flow in open channels. It is particularly desirable to determine pressure ratios in the various stages of the cycle.

For purposes of analysis, the "Comprex" cycle was divided into four parts--two compressions and two expansions. This thesis was restricted to the study of the compression processes. A compression wave in a pipe is analagous to a moving hydraulic jump in an open channel. A theoretical analysis of the hydraulic jump in a channel of triangular cross section was made and the equations of the analogy were extended to apply to this hydraulic jump. The equipment necessary to permit experimental verification of the theory was designed and constructed and a series of experiments was conducted.

Experimental results were found to be in good agreement with the simple one-dimensional theory.

It is recommended that future investigations of this nature be directed toward the study of expansion waves. An analysis of the expansion process in conjunction with the analysis of the compression process presented in this thesis would make possible the formulation of design criteria for the Comprex.

II. INTRODUCTION

The "Comprex"* is a pressure exchanger recently invented by Claude Seippel for the Brown Boveri Company of Baden, Switzerland. It is the first rotating machine to achieve the dual function of compressing and expanding gas in the same cylinder in a related sequence such that the cylinder walls attain a temperature equal to the mean of the mean temperatures of the working fluids. This latter condition permits incorporation of the "Comprex" in the Gas Turbine Cycle and results in increased efficiency of the Gas Turbine Cycle without resorting to high temperature alloys. An experimental study of the separate compressions and expansions in the "Comprex" would be extremely difficult. Because of the very high velocities to be considered, complicated measuring techniques and expensive equipment would be required. Professor A.H. Shapiro of the Mechanical Engineering Department suggested investigating the possibility of simplifying the experimental analysis of the cycle by application of the hydraulic analogy between gas flow in pipes and water flow in open channels. This suggestion led to the present investigation of the mechanics of the hydraulic jump in an open triangular channel.

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* U.S. Patent No. 2,399,394 dated 30 April 1946

Since the application of the hydraulic analogy to gas flow in the "Comprex" prompted this investigation, a short description of the cell mechanics of the "Comprex" is presented here.

The "Comprex" consists essentially of a simple cell rotor with a number of fixed, shrouded axial walls or vanes equally spaced around the shaft, and a housing with two entrance and two exit ports. Each cell alternately carries fresh air to be compressed and hot gas to be expanded--both flowing in the same direction. Referring to Figure I*, (1) low pressure fresh air enters the comprex cell, the exit port being closed; (2) a compression wave, traveling with a velocity nearly equal to the velocity of sound, shoots through the cell toward the entrance; (3) when the compression wave reaches the entrance, this port is closed and (4) the cell then contains air at an intermediate pressure with zero velocity; (5) the entrance port is then opened to high pressure gas and a second compression wave is produced, so that (6) high pressure gas, high pressure air and intermediate pressure gas exist in the cell; (7) when the second pressure wave reaches the exit port, that port is opened and high pressure air flows into the pressure space; (8) when sufficient high pressure

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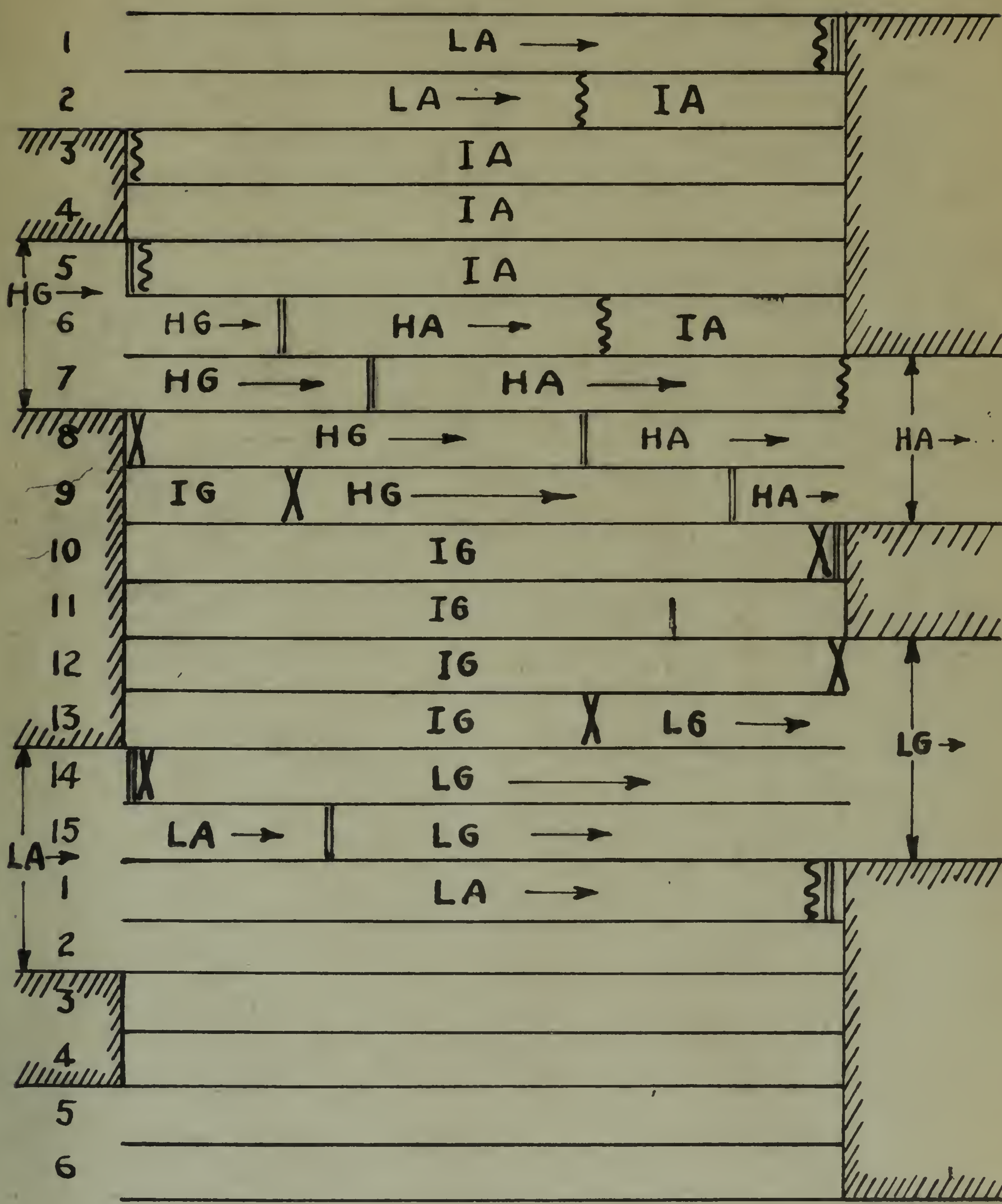
* This particular graphic representation of the cell mechanics of the "Comprex" must be credited to Professor A.H. Shapiro of the Mechanical Engineering Department.

FIGURE I

CELL MECHANICS OF THE COMPREX



LEGEND

A	Air	L	Low Pressure
G	Gas	‖	Gas-Air Boundary
H	High Pressure	§	Compression Wave
I	Intermediate Pressure	X	Expansion Wave

gas to be expanded has been introduced, the entrance port is closed; (9) this sudden interruption of flow causes an expansion wave to be generated; (10) when this expansion wave reaches the exit port, it is closed and (11) once again the cell contents are at an intermediate pressure (not necessarily the same pressure as the intermediate pressure air) and at rest. (12) The exit port is opened to a low pressure space and another expansion wave is generated (13) which shoots toward the entrance; (14) upon arrival of this wave at the entrance, its port is opened to low pressure fresh air, which displaces the low pressure gas flowing out, and the cycle repeats.

III. PROCEDURE AND DESCRIPTION OF EQUIPMENT

The equations for the analysis of a hydraulic jump in a channel of triangular cross section were derived. By means of these equations, the hydraulic jump was shown to be analogous to a compression wave in gas. Certain dimensionless parameters were plotted as functions of the Mach Number of the entering stream. In order to check the theory, the necessary equipment was designed, constructed, and set up in the basement of the Steam Engineering Laboratory of the Massachusetts Institute of Technology. The complete setup and its detailed construction may be seen clearly from the attached photographs and drawings. It consists of the following parts:

(1) The reservoir tank was four feet square and four feet high. It was made of 14 gage sheet steel and was of welded construction throughout. A 1 1/2 inch inlet pipe was provided at the rear. A gage glass and a 1 1/2 inch overflow line were located on one side near the front. The water level in the tank was controlled by adjusting the height of overflow outlet. The throttling gate was located at the center of the front face of the tank.

(2) The throttling gate consisted of a 3/8 inch steel plate, one foot square, mounted to slide up and down in a watertight seat. The gate opening was triangular. Thus by lifting the gate, the size of the triangular opening could be adjusted.

FIGURE II

RESERVOIR TANK

SCALE $\frac{3}{4}$" = 1 FT.

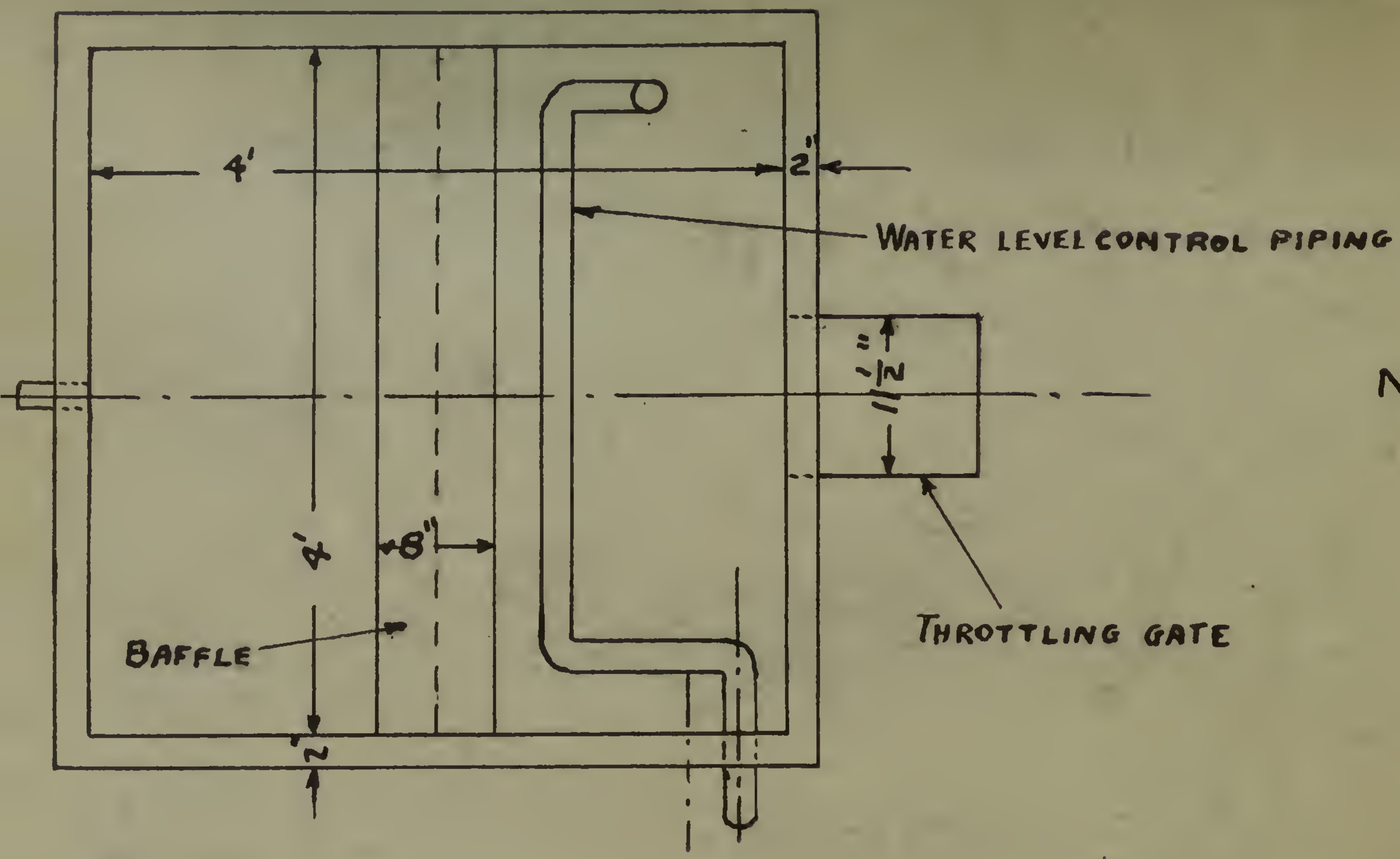
NOTE: 1. ALL PLATING IS 14 GAGE GALVANIZED STEEL UNLESS OTHERWISE INDICATED.

2. ALL PIPING IS $1\frac{1}{2}$" STANDARD STEEL PIPE.

3. WELDED CONSTRUCTION THROUGHOUT

4. FOR DETAILS OF THROTTLING GATE, SEE FIGURE III.

5. TANK SUPPORTED BY METAL FRAMEWORK 3'-3" HIGH.



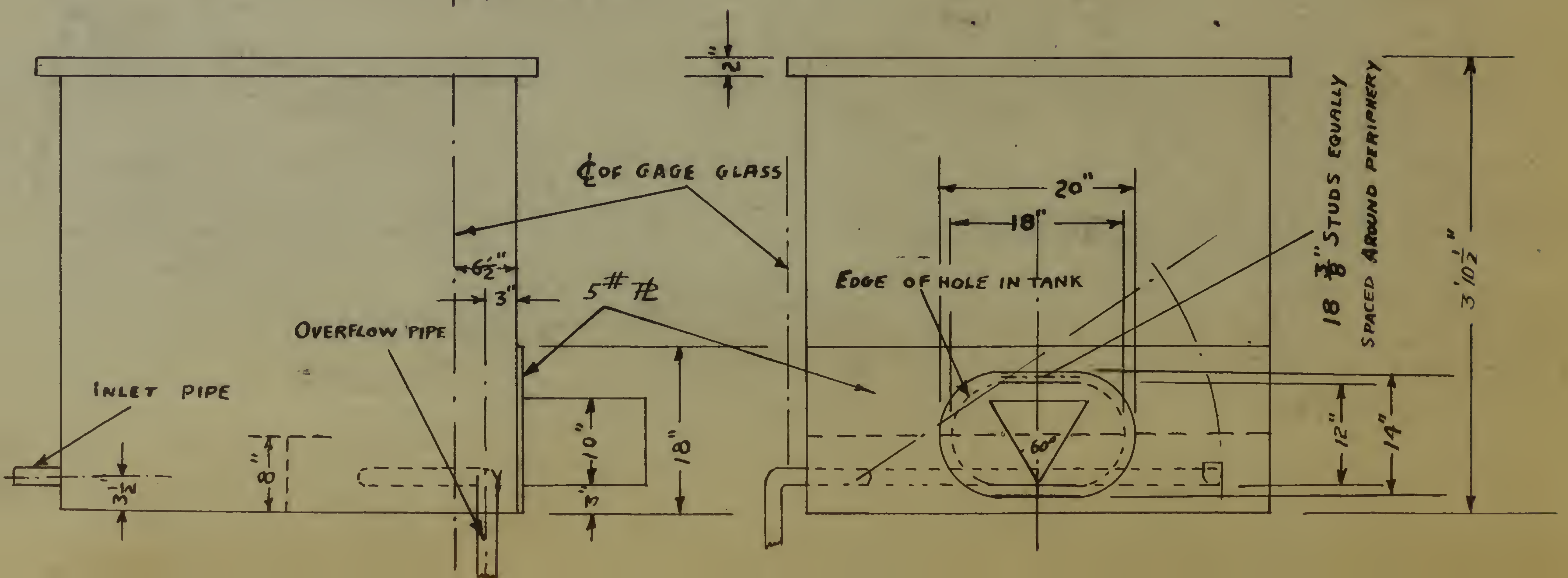
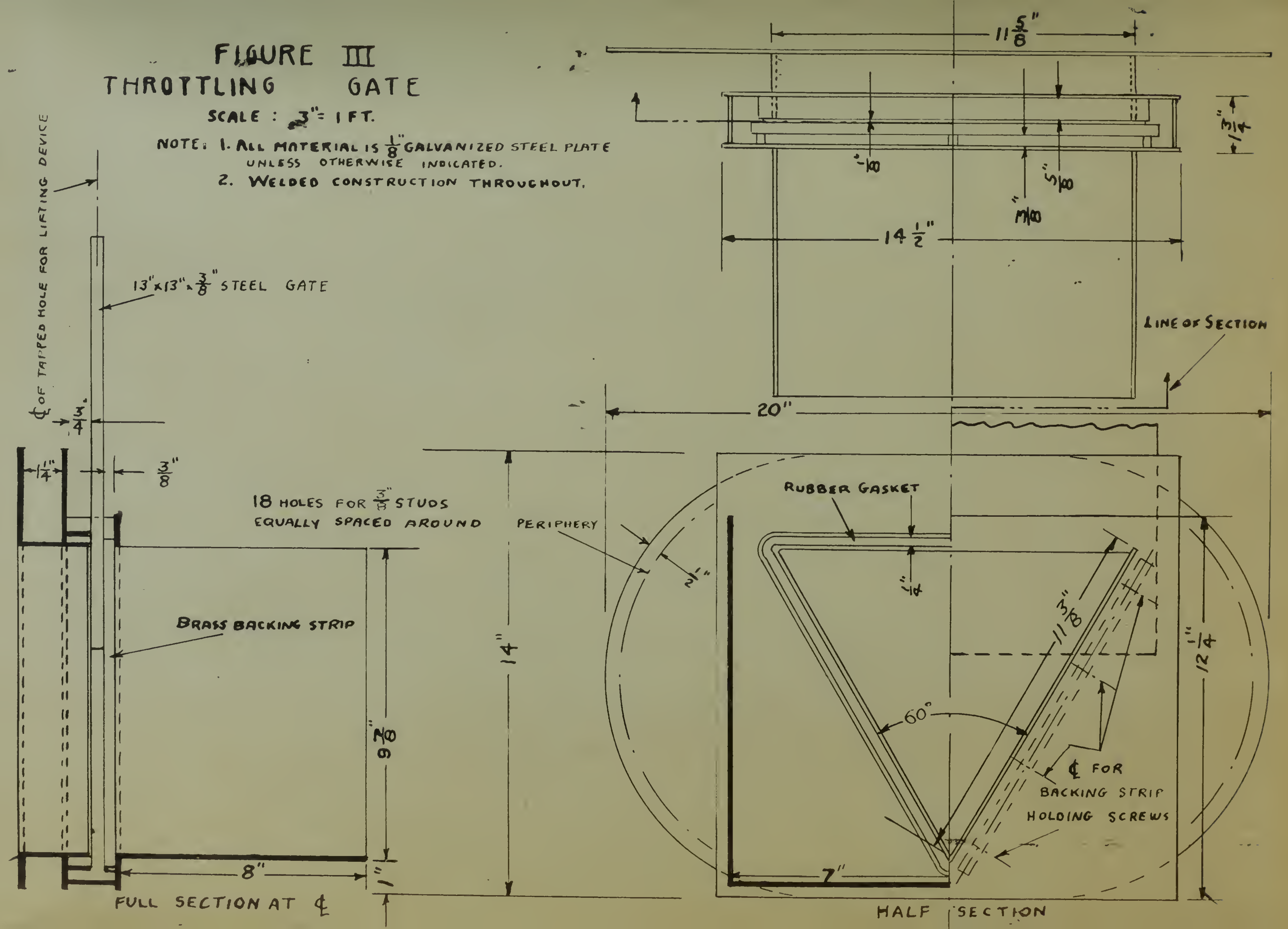


FIGURE III
THROTTLING GATE

SCALE : 3" = 1 FT.

NOTE: 1. ALL MATERIAL IS $\frac{1}{8}$" GALVANIZED STEEL PLATE UNLESS OTHERWISE INDICATED.
2. WELDED CONSTRUCTION THROUGHOUT.

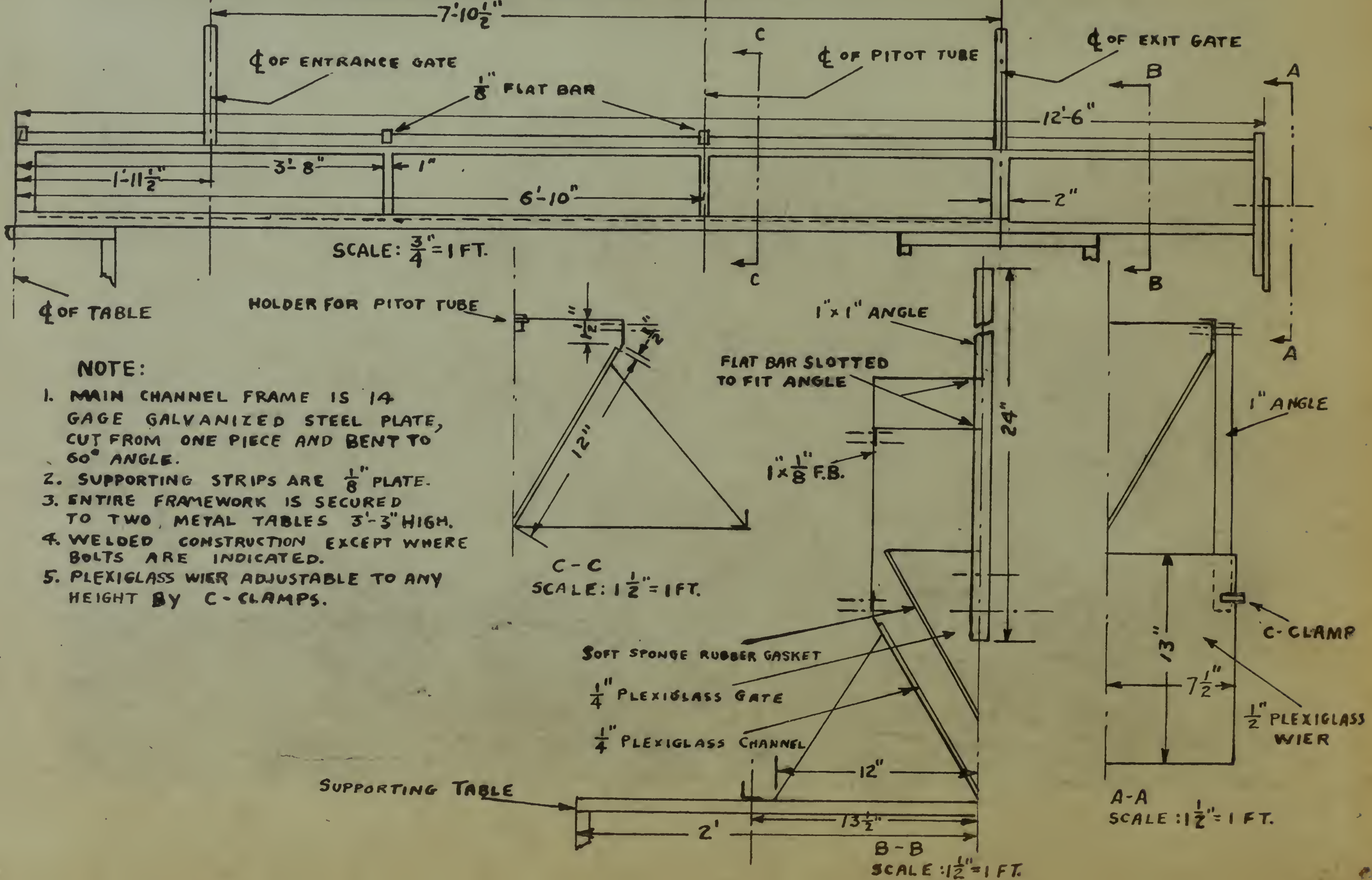


FULL SECTION AT ℄

HALF SECTION

FIGURE IV

CHANNEL AND GATING SYSTEM



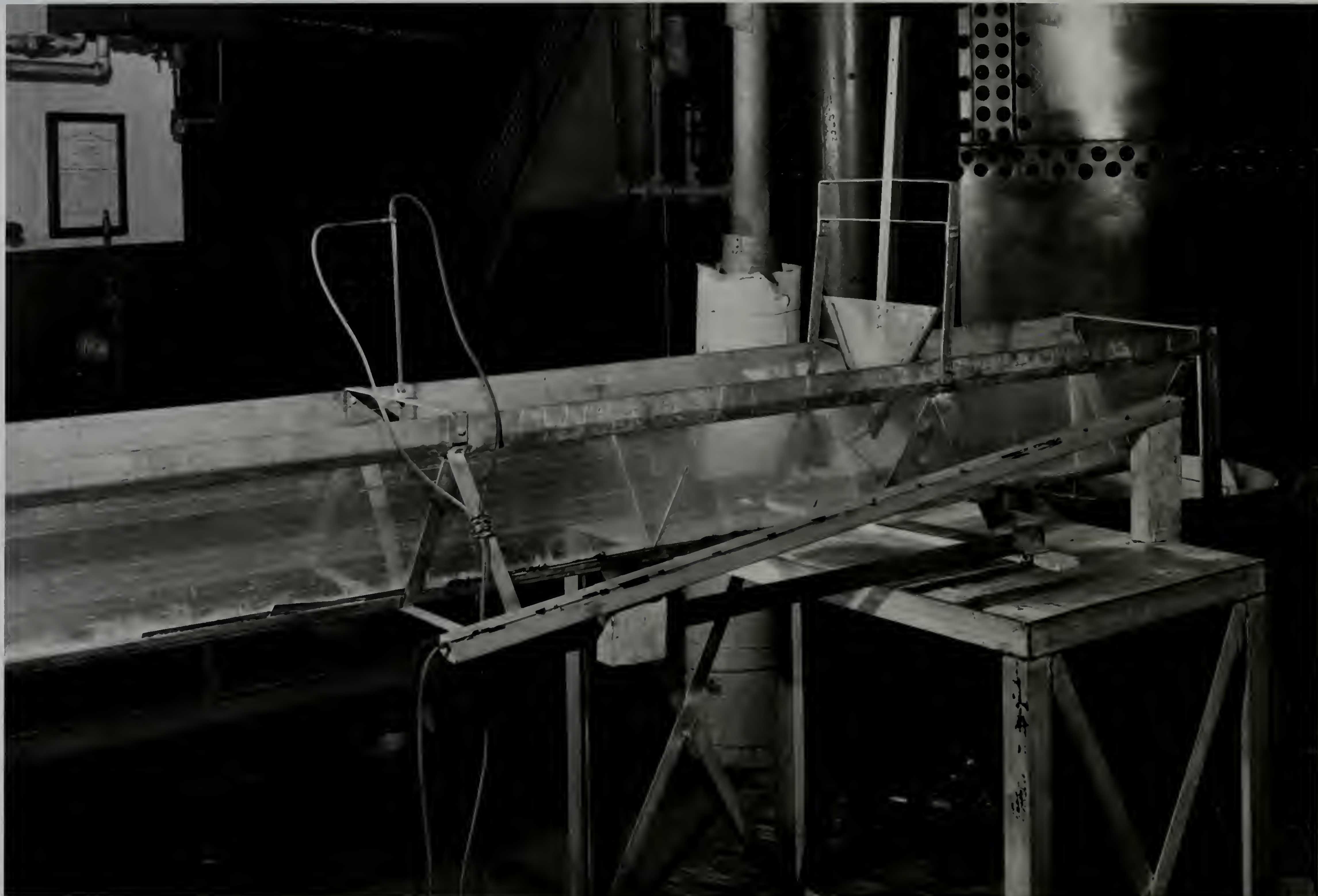
NOTE:

1. MAIN CHANNEL FRAME IS 14 GAGE GALVANIZED STEEL PLATE, CUT FROM ONE PIECE AND BENT TO 60° ANGLE.
2. SUPPORTING STRIPS ARE ⅛" PLATE.
3. ENTIRE FRAMEWORK IS SECURED TO TWO METAL TABLES 3'-3" HIGH.
4. WELDED CONSTRUCTION EXCEPT WHERE BOLTS ARE INDICATED.
5. PLEXIGLASS WIER ADJUSTABLE TO ANY HEIGHT BY C-CLAMPS.

FIGURE V

PHOTOGRAPHIC VIEW OF SET UP

FIGURE VI



PHOTOGRAPHIC VIEW OF PITOT TUBE AND SLANTED GAGE

(3) The triangular channel was twelve feet long and eleven inches deep with a 60° triangular cross section. The channel was made from 1/4 inch plexiglass sheets fitted and glued together with cellulose acetate. The plexiglass was supported by a sheet metal framework throughout its length. A plexiglass wier was mounted at the discharge end of the channel to provide for control of the velocity and depth of flow.

(4) The end gates were manufactured from 1/4 inch plexiglass and were shaped to fit the channel. A strip of 1/4 inch by 1/4 inch sponge rubber was glued to the edges of these gates to provide a watertight seal. Each gate was mounted on a two-foot section of one inch angle iron by which it could be raised and lowered. The angle iron rode in a vertical guide which prevented rotary motion of the gate.

(5) The circulating pump was a 3/4 horsepower U.S. Navy standard submersible pump. It took suction from the canal over which the equipment was mounted, and supplied water to the tank.

(6) The weighing tank was a cylindrical tank of about five cubic feet capacity. It was provided with a discharge line and a quick closing valve. The tank was placed upon a scale which was available in the laboratory.

Thus, for each run, the time required for a specific weight of water to flow could be measured.

(7) The pitot tube was a standard 1/4 inch pitot tube by which both static head and static plus velocity head could be measured. The two outlets were connected by 1/8 inch rubber tubing to 6 1/2 millimeter glass tubes. These tubes were five feet in length and were slanted to give a magnification of approximately 9 to 1. The tubes were calibrated throughout their length.

Experimental runs were made as follows:

Steady flow was established in the channel. The exit gate of the channel was then closed and a wave was generated which moved back toward the entrance. This wave can be considered as a moving hydraulic jump. As soon as the wave passed the inlet gate, this gate was closed, and the channel then contained water at a new, high level at zero velocity. The following quantities were measured:

1. The Mach Number, M_0, (defined as the ratio of the velocity of the stream to the velocity of propagation of small gravity waves in the same stream) for the water entering the channel.

2. The depth of the water before and after the wave.

3. The velocity of the wave with respect to the channel. (This velocity was measured by means of a stop watch.)

These experimental results were than compared with the theoretical analysis.

IV. RESULTS

The results of the analysis of a hydraulic jump in an open triangular channel are presented as plots of the various characteristic parameters vs. the Mach Number of the water entering the channel (M_0). All points plotted are those obtained by experiment. Figure VII is a plot of the ratio of the final depth of water after the hydraulic jump to the total head before the jump; first, for an observer stationary with respect to the channel; second, for an observer stationary with respect to the jump. Figure VIII and Figure IX show the characteristic parameters for the hydraulic jump. Figure X shows how the efficiency of the two compressions in series, for a given overall compression ratio, will vary as the ratio between the separate compression ratios is varied. Figure XI shows efficiency curves for two compressions in series for two conditions, first, for equal compression ratios; second, for equal stream velocities. Figure XII shows the characteristic parameters for two compressions in series for two conditions; first, for equal compression ratios; second, for equal stream velocities.

Photographs of the hydraulic jump and of expansion waves were taken through the side of the plexiglass channel. The side of the channel away from the camera was covered

FIGURE VII

TOTAL HEAD RATIOS FOR STATIONARY AND MOVING HYDRAULIC JUMPS

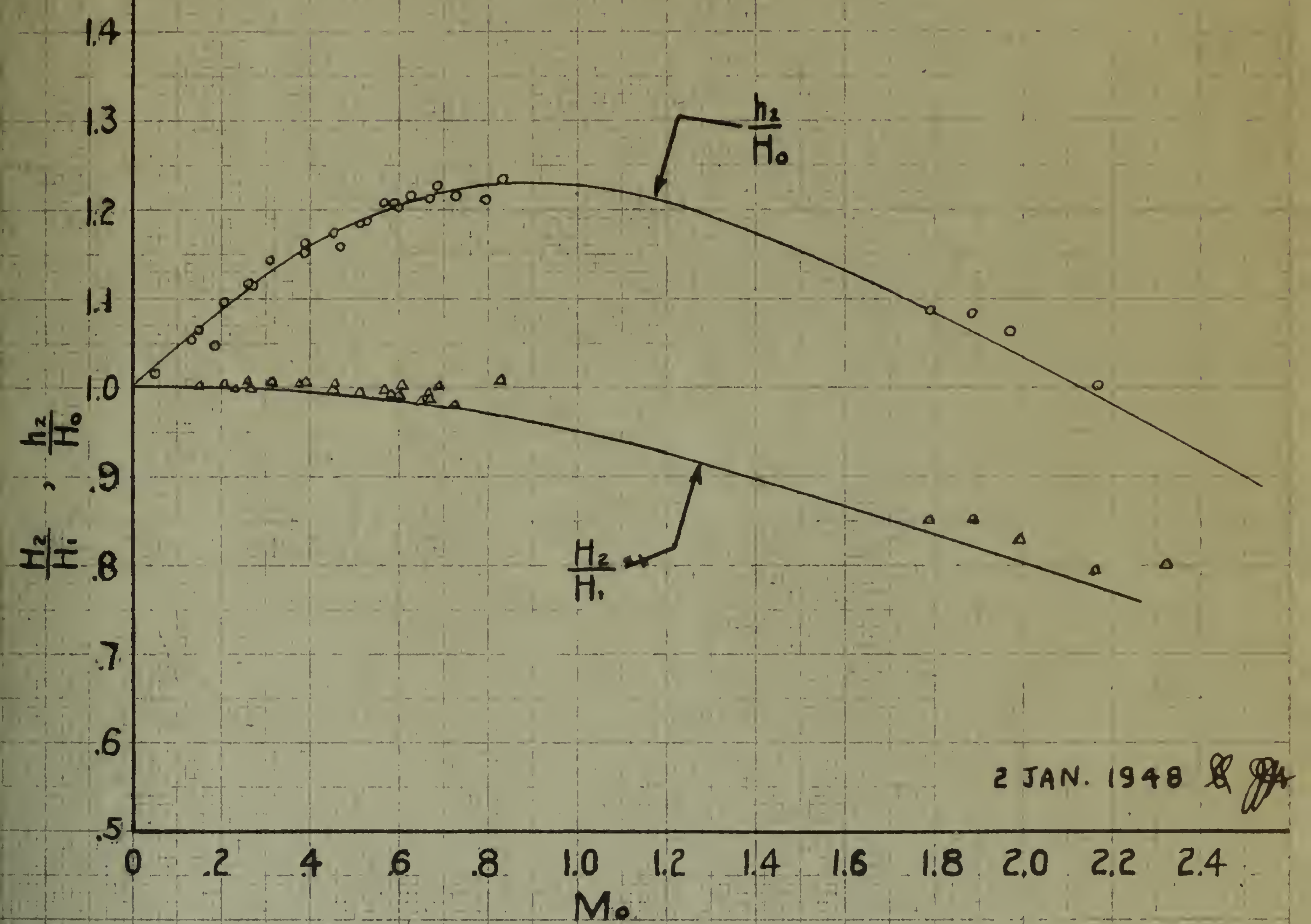
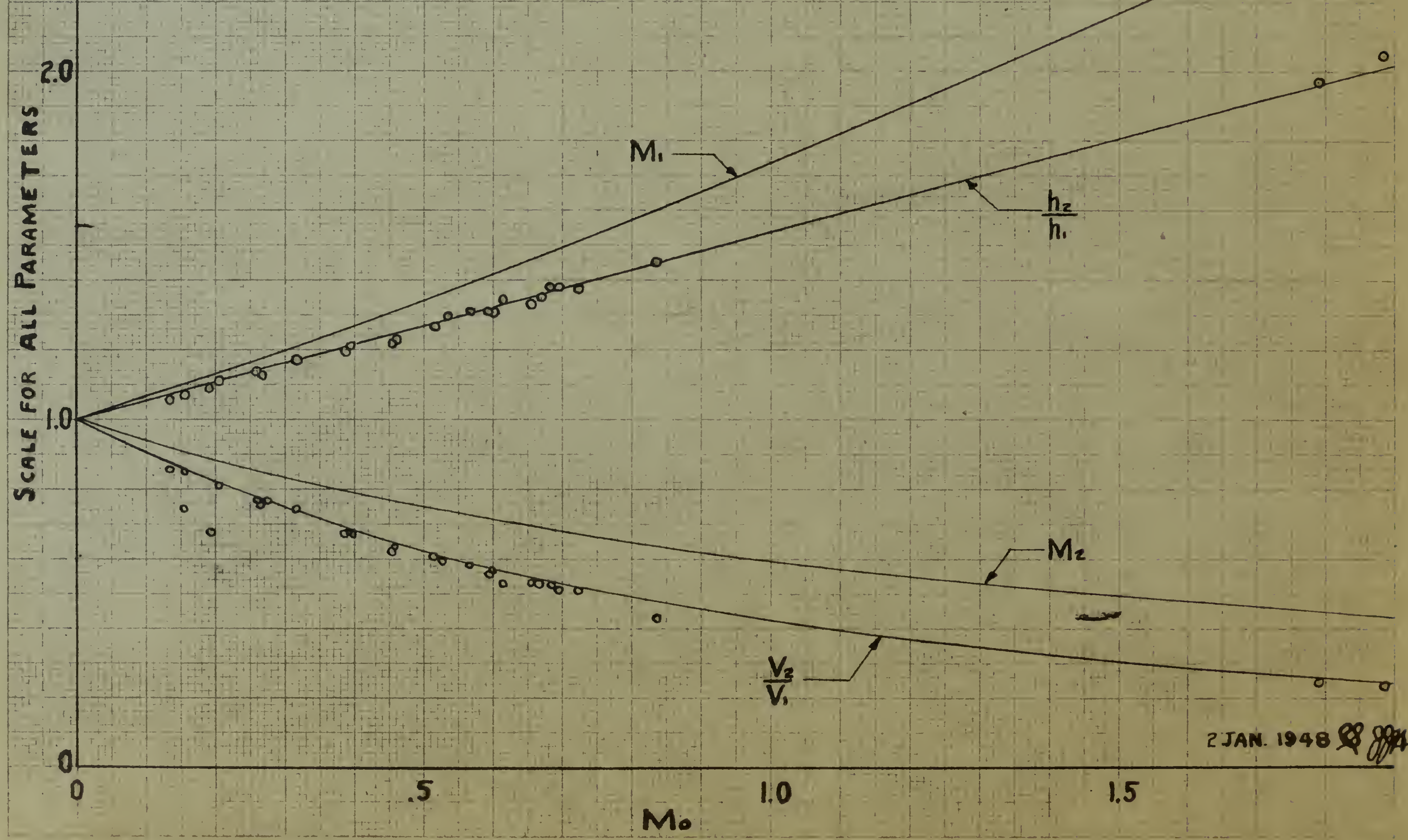


FIGURE VIII
CHARACTERISTIC PARAMETERS FOR
THE HYDRAULIC JUMP



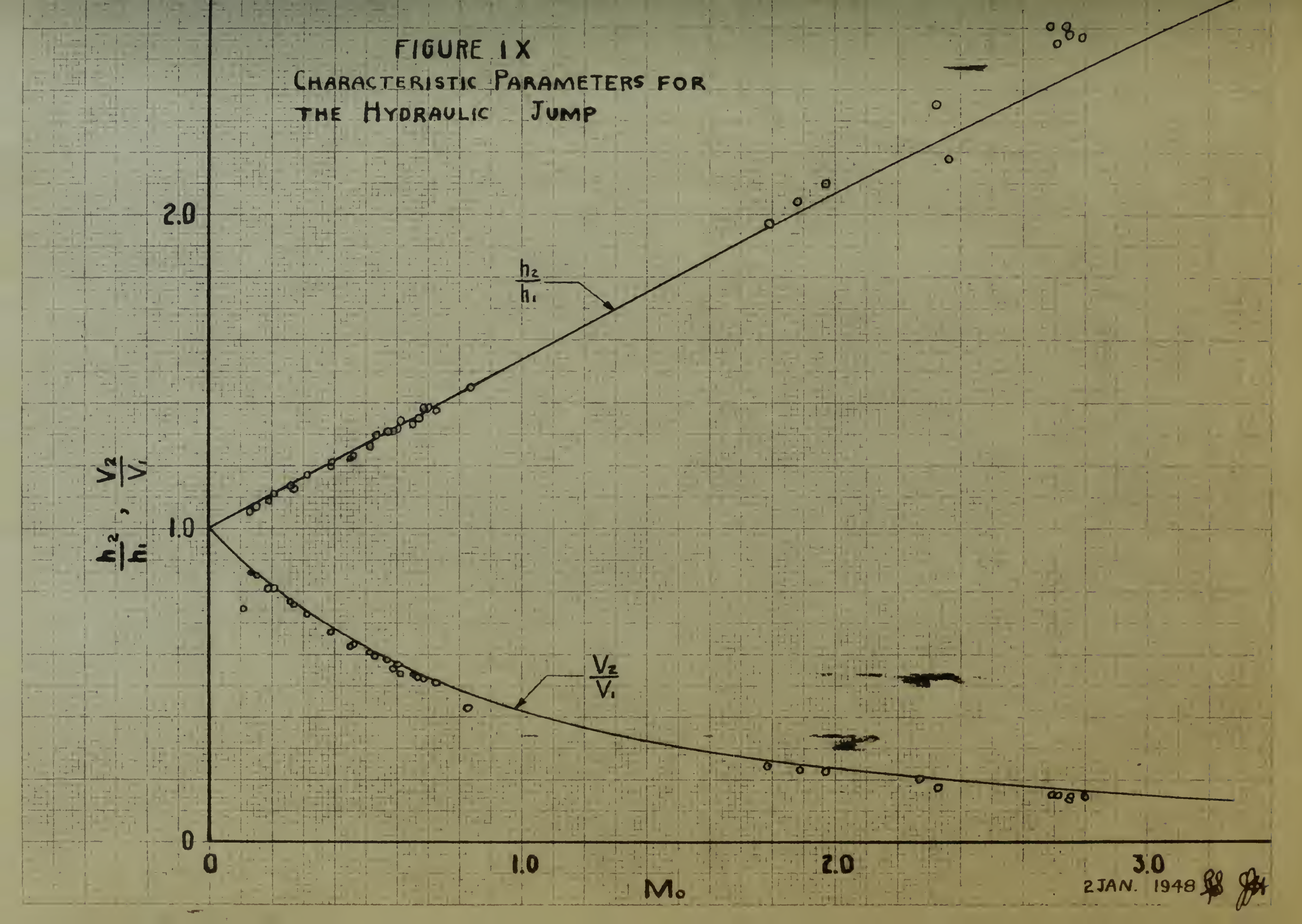


FIGURE IX
CHARACTERISTIC PARAMETERS FOR THE HYDRAULIC JUMP
$\frac{h_2}{h_1}$, $\frac{V_2}{V_1}$
2.0
1.0
0
$\frac{h_2}{h_1}$
$\frac{V_2}{V_1}$
0
1.0
2.0
3.0
M_0
2 JAN. 1948

FIGURE X

EFFICIENCY CURVES FOR CONSTANT OVERALL COMPRESSION RATIO

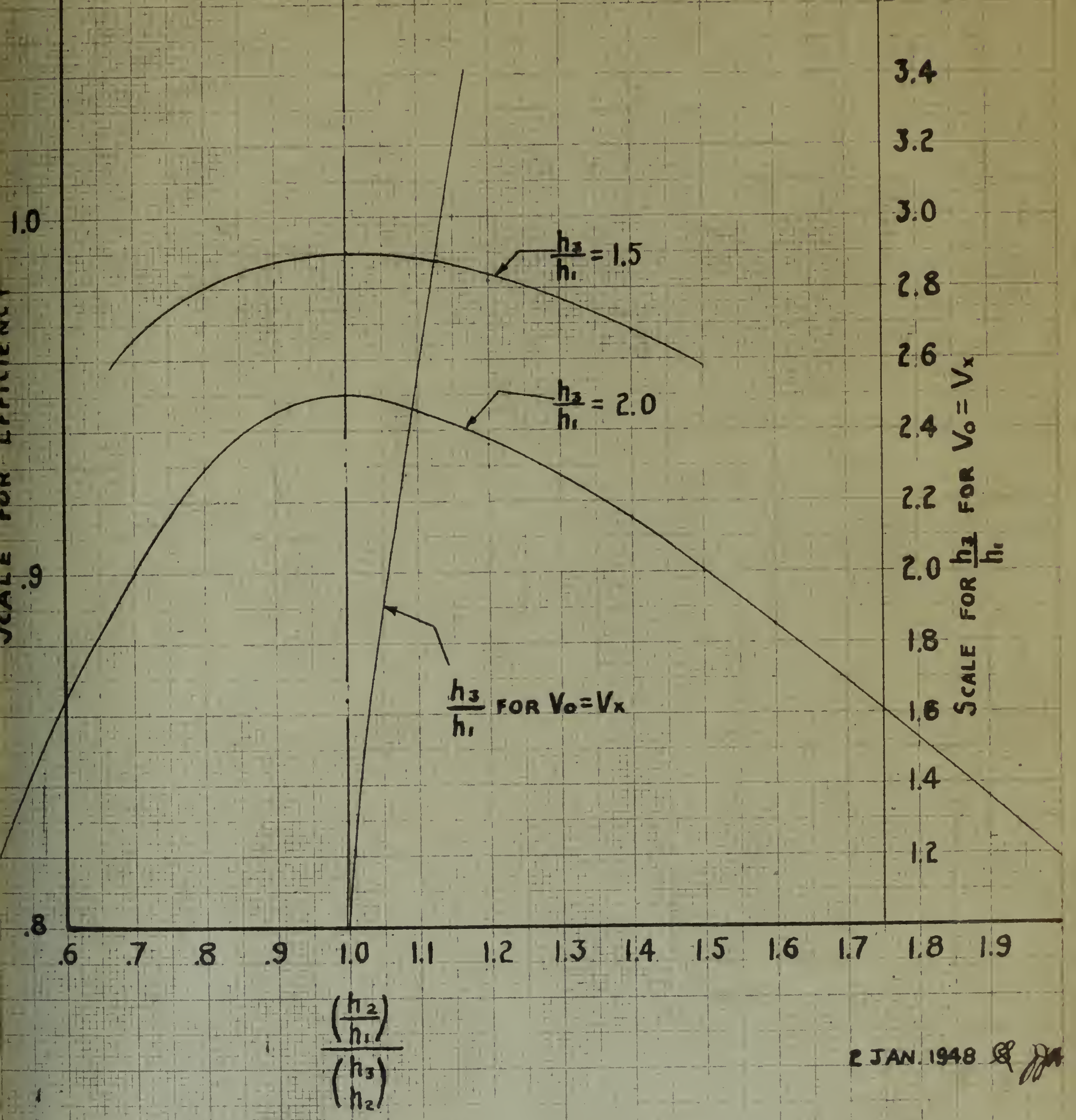


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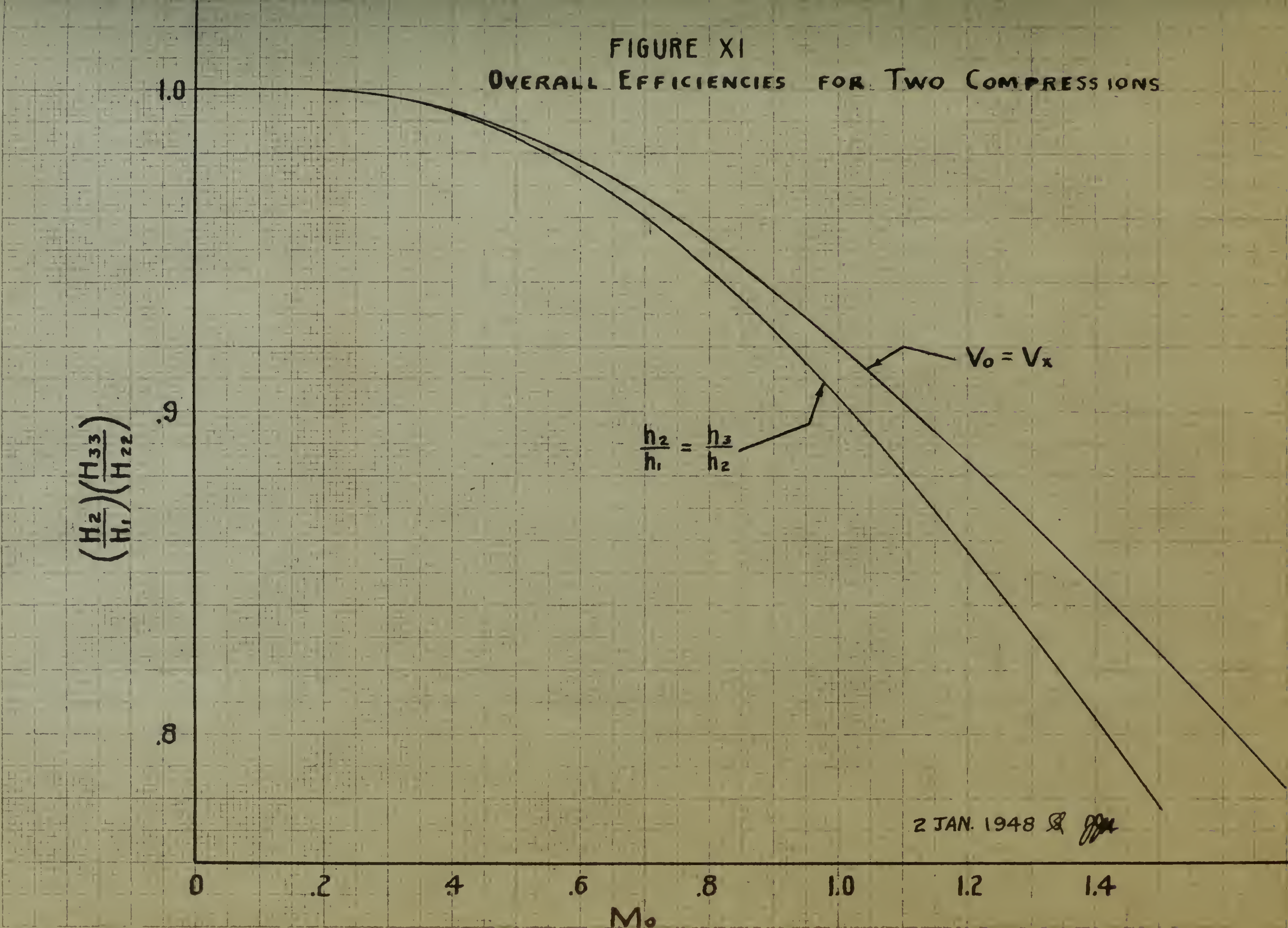
FIGURE XI

OVERALL EFFICIENCIES FOR TWO COMPRESSIONS



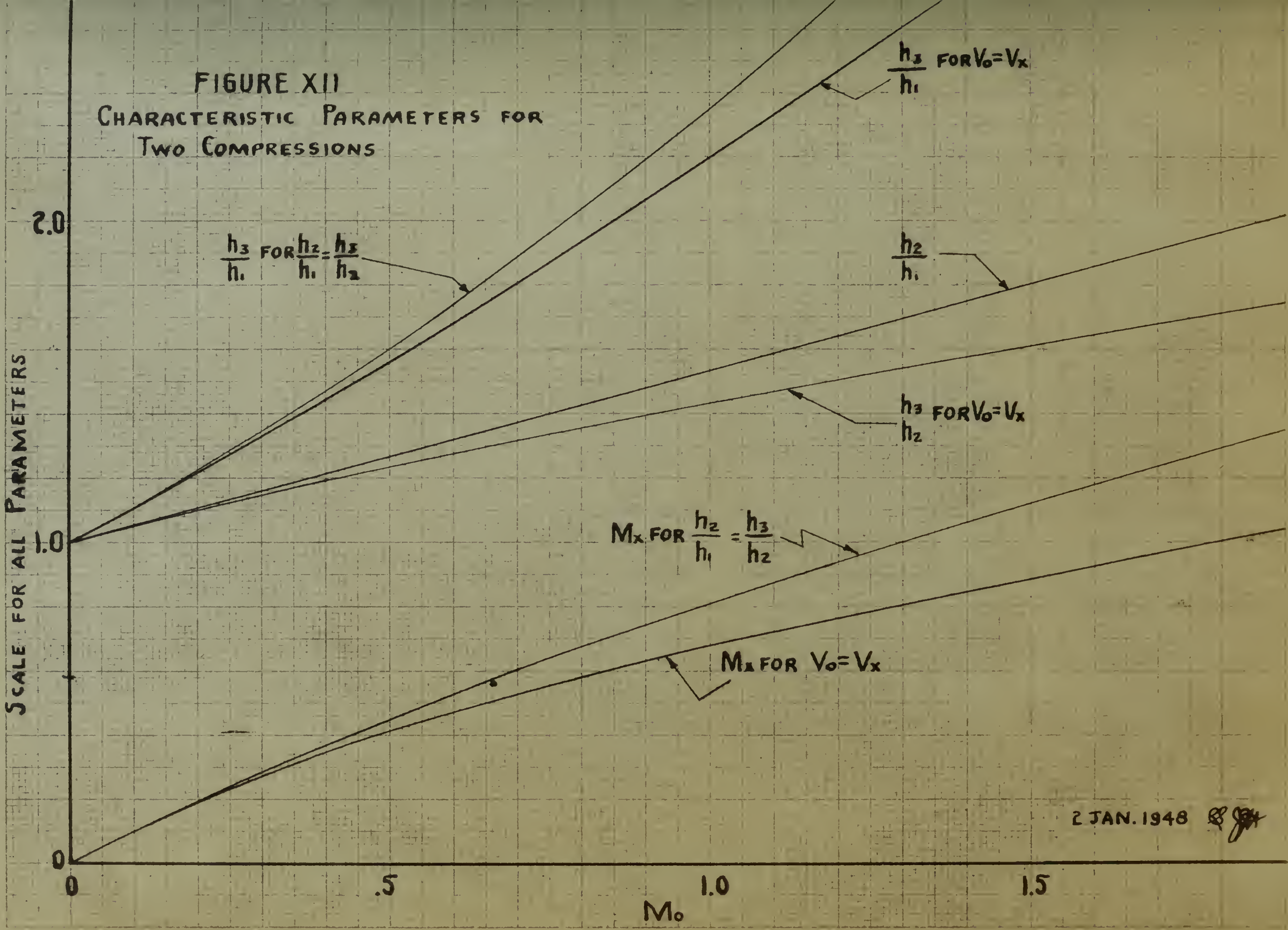


FIGURE XII
CHARACTERISTIC PARAMETERS FOR TWO COMPRESSIONS

with a single thickness of plain white paper; two flood lights were placed about eighteen inches in back of the paper-covered side; and photographs were taken at a distance of three feet from the near side. The sharp line in each photograph is a trace of the water surface on the channel wall. Photographs of the hydraulic jump were also made from directly above the channel. Both sides of the channel were covered with a single thickness of paper and the two flood lights placed about eighteen inches below the channel. In these photographs, the dark area in the center of the channel is the angle iron support at the bottom of the channel. All photographs were taken with a Dollina - 35 mm. camera using an f-4.5 setting at 1/100 of a second. The following series shows the nature of compression and expansion waves in a triangular channel.

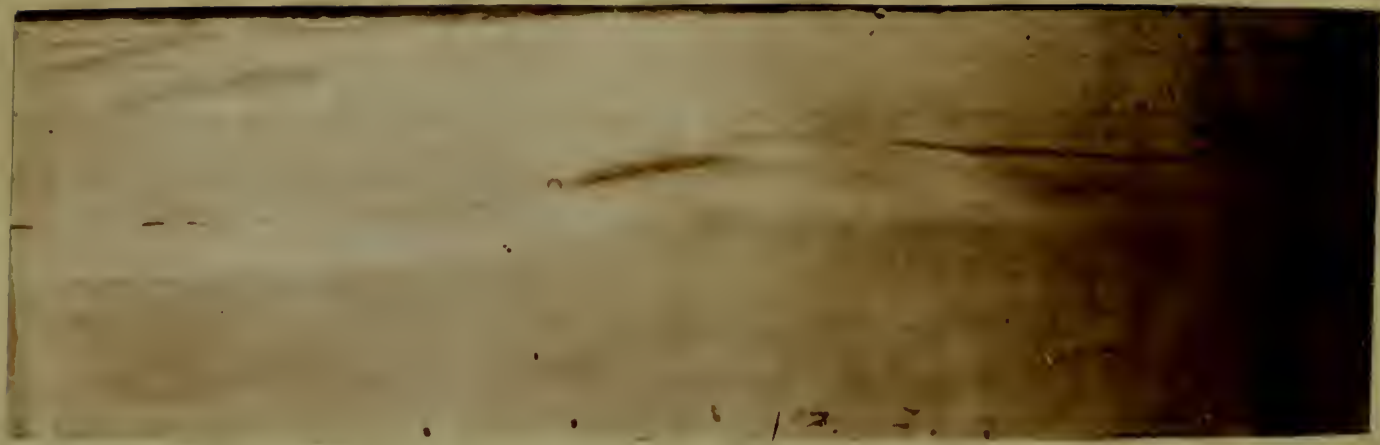


FIGURE XIII

FIGURE XIV

FIGURE XV

Compression wave photographed for three degrees of development showing how wave front develops into hydraulic jump as wave travels along channel.

FIGURE XVI

FIGURE XVII

FIGURE XVIII

Expansion wave photographed for three degrees of development showing how wave shape attenuates as wave travels along channel.

FIGURE XIX

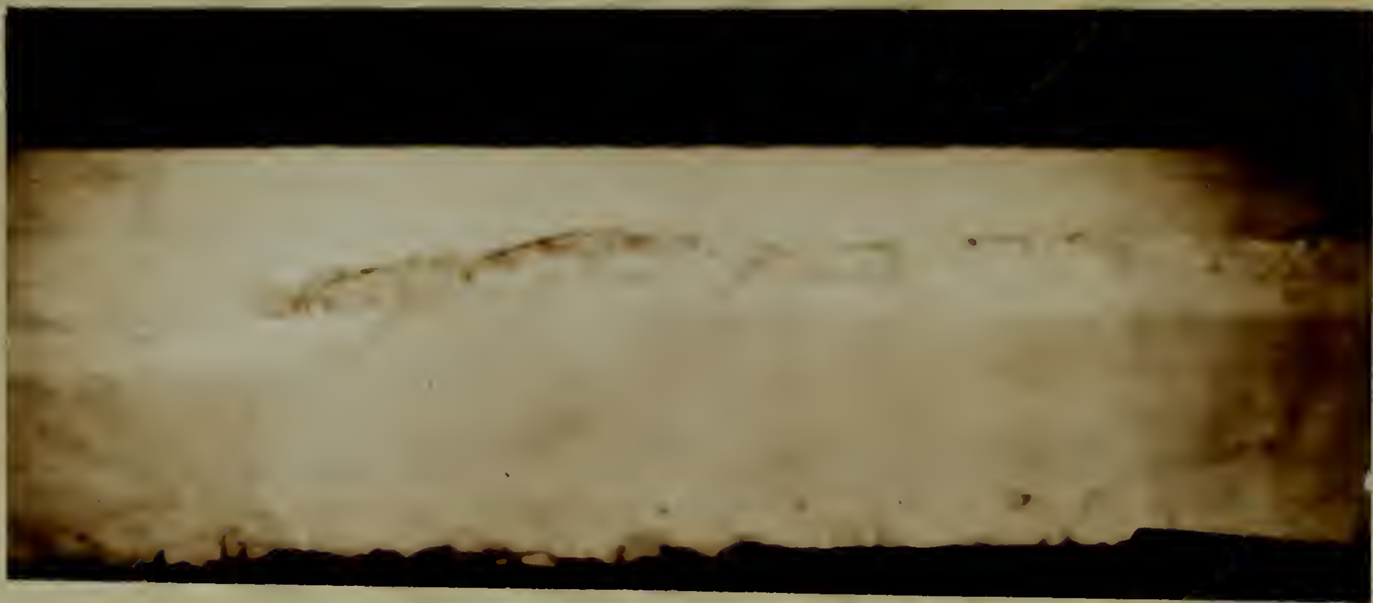


FIGURE XX

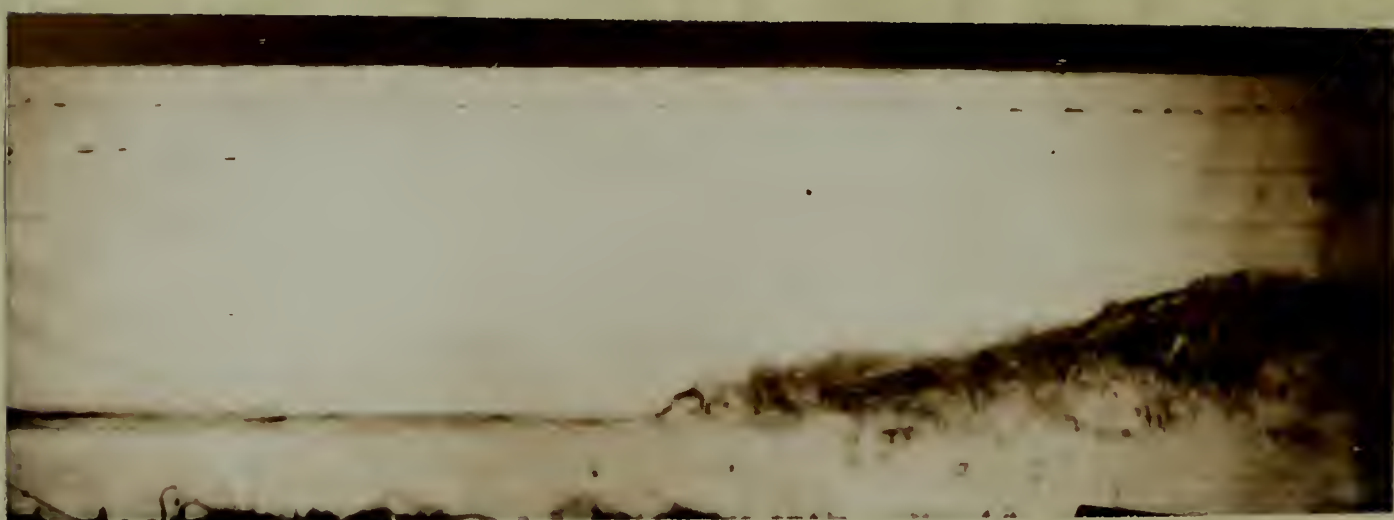


FIGURE XXI

Side view of compression waves in stream with Mach Numbers of 0.45, 0.72, and 2.75, respectively.

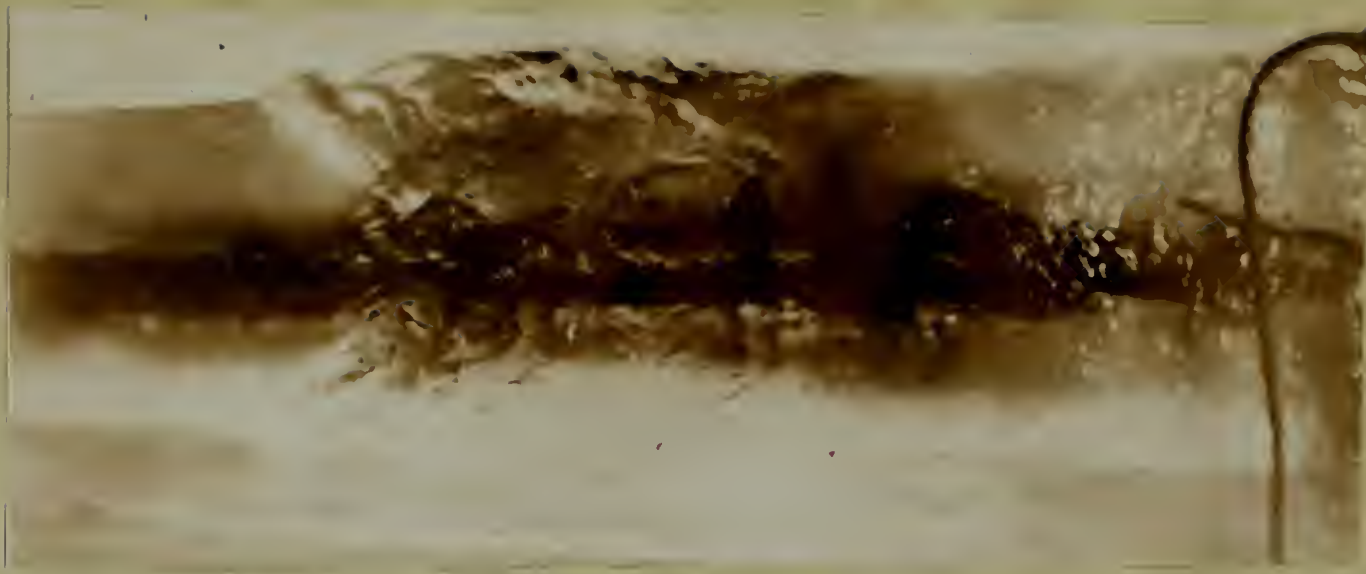


FIGURE XXII

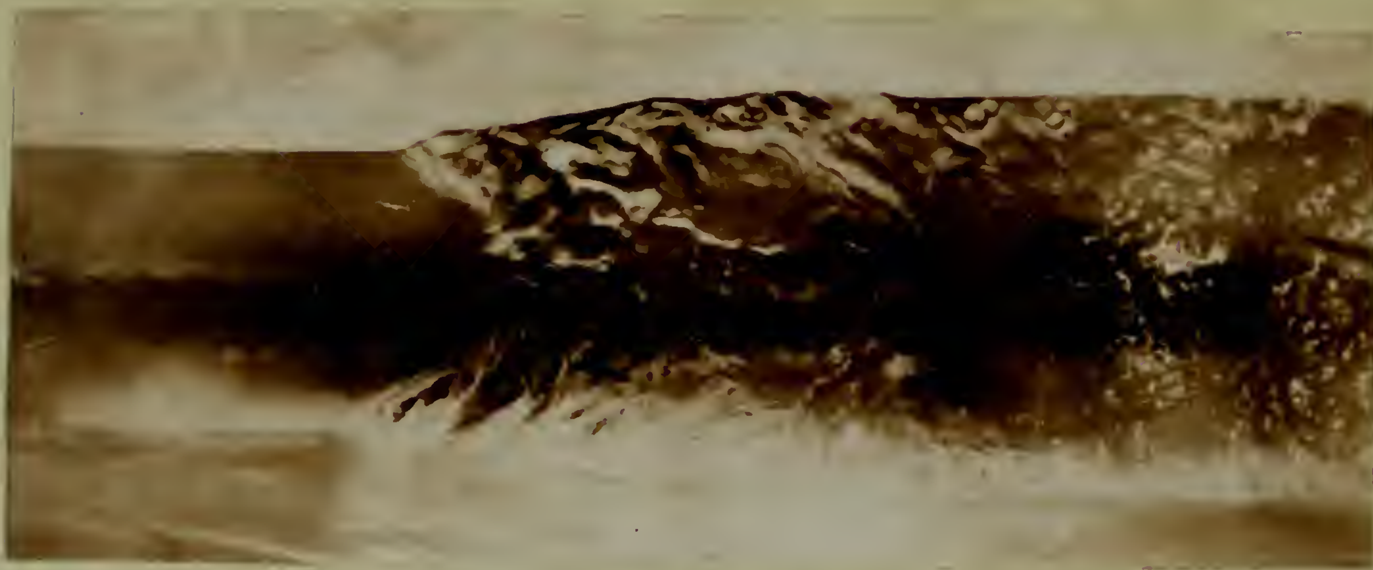


FIGURE XXIII

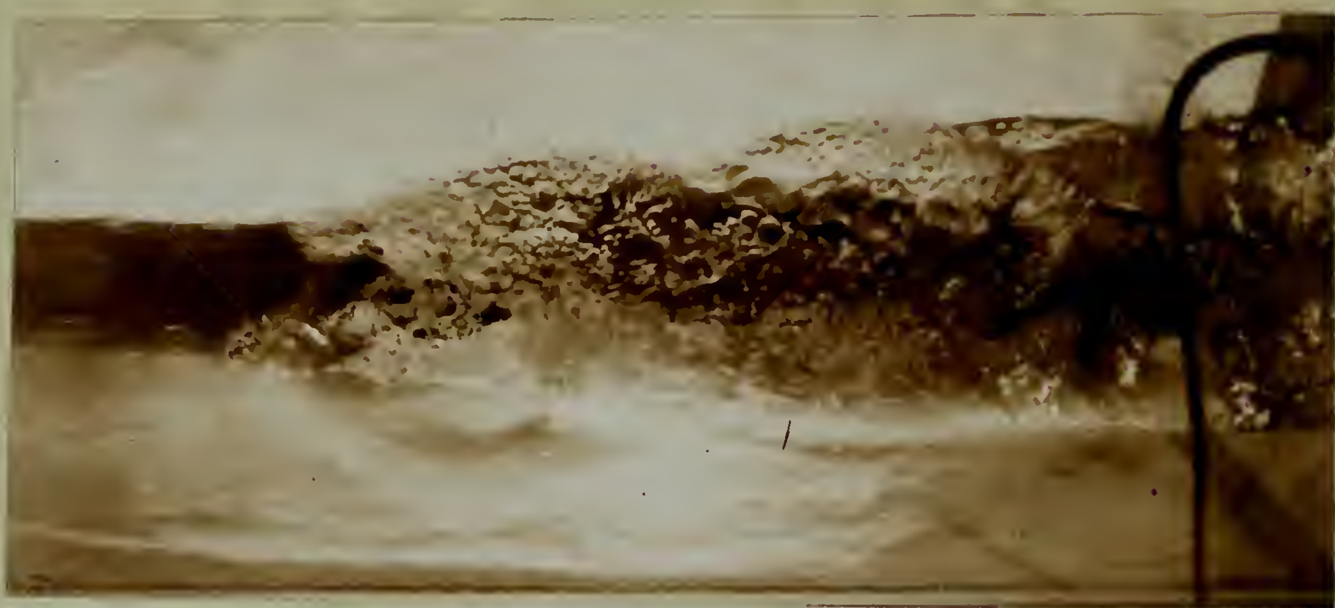


FIGURE XXIV

Top view of compression waves in streams with Mach Numbers of 0.45, 0.72, and 2.75, respectively.

FIGURE XXV

FIGURE XXVI

Top view of compression waves in streams of subcritical and supercritical flow, respectively, showing differences in wave length, wave shape, and degree of turbulence.

V. DISCUSSION OF RESULTS

A study of the results leads to the primary conclusion that experiment agrees very closely with the simple one-dimensional analysis of the hydraulic jump in a triangular channel.

From the theoretical analysis the following unusual effect can be pointed out. The depth of the water in the channel, after the generated wave has passed, can be greater than the total head of the entering stream. This, at first, appears to be a violation of the Second Law of Thermodynamics. But it must be remembered that this is not a steady-flow process, and the ratio of the final depth to the initial total head can exceed unity. It is shown in Figure VII that this ratio reaches a maximum of 1.23 at a Mach Number of the entering stream equal to about 0.87.

Suppose now that the observer is moving with the velocity of the wave. The wave becomes a stationary hydraulic jump, and we have steady state conditions. From a study of these conditions, the following conclusions can be drawn: (1) It is seen from Figure VII that the total head after the jump is always less than the total head before the jump. (2) The depth ratios, velocity ratios, and total head ratios through the jump are a function only of the Mach Number of the entering steam, M_0. These are plotted in Figure VIII

and IX. (3) It is evident from the photographs of compression and expansion waves that expansion waves cannot be treated as reverse hydraulic jumps. The wave shape is continually changing and the analysis applied to the compression wave is not applicable.

If, after the first compression takes place, we open the inlet gate of the channel and put the stationary water in the channel in contact with water at a higher head, a wave will be generated and will move down the channel toward the exit gate. Water behind the wave will have a certain stream velocity V_x and a depth, h_3, greater than the depth of the water in front of the wave. This wave can also be considered as a moving hydraulic jump and is analagous to the second compression wave in the Comprex. For an observer moving with the velocity of the wave, i.e., steady state conditions, the wave becomes a true hydraulic jump. The analysis developed for the first compression can be applied directly to this second compression. If the two compressions are considered to occur in series, we have what amounts to a water analogue for the two compressions in the Comprex.

For a given overall compression ratio, the results show that for optimum conditions, the first and second compression ratios should be equal. In the Comprex, in order to eliminate the need for turning vanes and the

the resulting losses and mechanical complications, it is desirable to design for the condition that the velocities of the two entering streams are equal ($V_0 = V_x$). Let us determine how closely the condition for equal stream velocities approximates the optimum condition.

In this analysis, the ratio of final total head to initial total head for the stationary hydraulic jump is considered as a measure of the efficiency of the jump. The product of these ratios for the first and second compression may be considered a measure of the overall efficiency of the two compressions. For any given value of the overall compression ratio $\frac{h_3}{h_1}$, this product $\left(\frac{H_2}{H_1}\right)\left(\frac{H_{33}}{H_{22}}\right)$ may be plotted as a function of the ratio of the first compression ratio to the second compression ratio $\frac{h_2/h_1}{h_3/h_2}$

When the two compression ratios are equal, this parameter becomes unity, and the efficiency is at its maximum value. Proof of this is given in appendix "B". Plots of the efficiency for two values of the overall compression ratio are shown in Figure X. This figure also shows a plot of the parameter $\frac{h_2/h_1}{h_3/h_2}$ for the condition of equal stream velocities for varying values of the overall compression ratio $\frac{h_3}{h_1}$. The value of this parameter

departs only slightly from unity (the optimum condition) for all reasonable values of the overall compression ratio $\frac{h_3}{h_1}$. Since the efficiency curves are very flat in the vicinity of the maximum value, it appears the variations from the optimum condition caused by adoption of equal stream velocities will have negligible effect. Figure XI is a plot of the important parameters for the optimum condition (equal compression ratios) and for the condition of equal stream velocities. Figure XII compares the efficiencies for the same two conditions. All the above values are plotted vs. the Mach Number of the entering stream (M_o). It is seen that for a given M_o, a higher overall compression ratio is obtained by using equal compression ratios than by using equal stream velocities. This higher compression ratio is accompanied by a slightly lower efficiency, for a given M_o, so that the gain apparent in the figure is actually not quite as great as it at first seems. Differences in the performance for the two conditions are negligible.

If the Mach Number, velocity and density of the low pressure stream entering the "Comprex" are specified, it is necessary to specify only two of the corresponding variables for the entering high pressure stream in order to fix all states in the complete cycle. Referring to Figure I: The fluid at intermediate head (step 11) must be at such a state that the low pressure gas leaving the unit will have the same

velocity and head as the entering low pressure air. This is necessary to prevent mixing and secondary waves. Similarly the state of the entering high level fluid must be such that it will expand to the proper intermediate state. Thus the relationship between the Mach Number of the high pressure stream and its velocity is fixed. If we set $V_x = V_o$, say, then all states in the complete cycle are specified; but an analysis of the complete cycle cannot be made without knowledge of the characteristics of the expansion process. This points out the importance of further investigation of the expansion processes.

Preliminary calculation of experimental data showed very good agreement with theory. Subsequent investigation indicated that three corrections should be made to velocities based on flow rate:

(1) The angle of "V" of the channel was found to be 62.5° instead of 60° as designed. The velocity for a given flow rate decreases as the angle increases.

(2) The water in the canal in the steam laboratory is Charles River water in which there is usually a sufficient amount of lubricating oil to cause some variation in density. The average specific gravity of the samples tested was .980. Although there may have been some variation from this figure throughout the experiments, this

value was used in the calculations.

3. The velocity of impact of the water stream on the weighing tank decreased as the tank filled. This differential in impact velocity caused a corresponding force differential on the scale. This force differential was sufficient to cause an error of 0.5 to 1.5 pounds in 200 pounds. Since the range of error and percentage error are small, it was decided to correct all flow rates on the basis of 1.0 pound in 200 pounds.

The net effect of the above three corrections is on the order of 2.5%. Since the majority of results are plotted against M_o (a function of this measured velocity), it was deemed desirable and necessary to include these corrections for best accuracy.

It is believed that Mach Numbers measured and corrected in the above manner are accurate within one per cent. Of all the quantities measured, the wave velocity must be considered to be the least accurate. An error of 0.2 seconds in 6 seconds is not unreasonable. This corresponds to an error of about 3 per cent. All other measurements are believed to be accurate within one per cent with the single exception of the static heads of the entering stream when M_o exceeded unity. In the latter case, the pitot tube does not indicate correctly, so

measurements were made by lowering a meter stick from a known distance above the bottom of the channel to the water surface. This method of determining static head cannot be considered to be accurate to within more than [illegible]% because of the turbulent nature of the surface when $M_o > 1.0$.

APPENDIX "A"

THE HYDRAULIC ANALOGY

The extension of the hydraulic analogy to application to a channel with a function of cross sectional area $Z = y^n$ was made by Loh (1). Some of the results of his analysis are presented here for the sake of completeness, without proof and with the notation used in this thesis.

The basic assumptions used throughout the whole analysis are:

(a) The fluid is frictionless.

(b) The flow is one dimensional and is in a pipe or channel of uniform cross section.

Present Notation

$$\frac{\rho_{a2}}{\rho_{a1}} = \left(\frac{h_2}{h_1}\right)^2$$

$$\frac{V_a}{a} = \frac{V}{\sqrt{\frac{gh}{2}}}$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1}$$

$$\frac{p_2}{p_1} = \left(\frac{h_2}{h_1}\right)^3$$

$$k = 1.5$$

Thus, flow of water in a triangular channel with a horizontal bottom is analogous to flow in a pipe of gas with $k = 1.5$.

Certain other relationships which will prove useful are developed here.

The wave propagation velocity of water in an open channel of a constant cross section of arbitrary shape with a straight horizontal bottom was derived mathematically by J. McCowan (4).

$$V_g = \sqrt{\frac{gA}{b}} \tag{6}$$

where A = area of the cross section

b = breadth of the free surface

For a triangular section:

$$V_g = \sqrt{\frac{gh}{2}} \tag{7}$$

The ratio, $\dfrac{V}{\sqrt{gh/2}}$ is analagous to the Mach Number of gas and will hereafter be referred to as the Mach Number for water.

$$M_1 = \frac{V}{\sqrt{\frac{gh}{2}}} = \frac{V_a}{a} \tag{8}$$

The ratio of the total head after compression to the total head before compression is

$$\frac{h_2}{H_0} = \frac{h_2}{\frac{V_0^2}{2g} + h_1} = \frac{h_2/h_1}{\frac{M_0^2}{4} + 1} \tag{9}$$

For a gas:

$$\frac{p_1}{p_0} = \frac{1}{\left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}}} \tag{10}$$

If k = 1.5,

$$\frac{p_1}{p_0} = \frac{1}{\left[\frac{M_1^2}{4} + 1\right]^3} \tag{11}$$

$$\left(\frac{p_1}{p_0}\right)\left(\frac{p_2}{p_1}\right) = \frac{p_2}{p_0} = \frac{\frac{p_2}{p_1}}{\left[1 + \frac{M_1^2}{4}\right]^3} \tag{12}$$

Hence, since,

$$\frac{p_2}{p_1} = \left(\frac{h_2}{h_1}\right)^3 \tag{4}$$

$$\frac{p_2}{p_{01}} = \left(\frac{h_2}{H_0}\right)^3 \tag{13}$$

We may also write:

$$\frac{M_2}{M_1} = \frac{\frac{V_2}{V_1}}{\sqrt{\frac{h_2}{h_1}}} = \frac{\frac{V_{a2}}{V_{a1}}}{\frac{a_2}{a_1}} \tag{14}$$

$$a = K\sqrt{T} \tag{15}$$

hence:

$$\frac{a_2}{a_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{h_2}{h_1}} \tag{16}$$

Therefore:

$$\frac{V_2}{V_1} = \frac{V_{a2}}{V_{a1}} \tag{17}$$

Since

$$\frac{p_{o1}}{p_1} = \left[\frac{M_1^2}{4} + 1\right]^3 \tag{18}$$

then

$$\frac{p_{o2}}{p_{o1}} = \frac{p_2}{p_1}\,\frac{\left[\frac{M_2^2}{4} + 1\right]}{\left[\frac{M_1^2}{4} + 1\right]} \tag{19}$$

From the theoretical analysis:

$$\left(\frac{H_2}{H_1}\right)^3 = \left(\frac{h_2}{h_1}\right)^3 \frac{\left[\frac{M_2^2}{4} + 1\right]}{\left[\frac{M_1^2}{4} + 1\right]} \tag{42}$$

hence:

$$\frac{p_{o2}}{p_{o1}} = \left(\frac{H_2}{H_1}\right)^3 \tag{20}$$

Summarizing the results of Loh's analysis and of the present analysis:

For a triangular channel:

$$k = 1.5 \tag{5}$$

$$M_{gas} = M_{water} \tag{2}$$

$$\frac{V_{a2}}{V_{a1}} = \frac{V_2}{V_1} \tag{17}$$

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} \tag{3}$$

$$\frac{\rho_{a2}}{\rho_{a1}} = \left(\frac{h_2}{h_1}\right)^2 \tag{1}$$

$$\frac{p_2}{p_1} = \left(\frac{h_2}{h_1}\right)^3 \tag{4}$$

$$\frac{p_2}{p_{01}} = \left(\frac{h_2}{H_0}\right)^3 \tag{19}$$

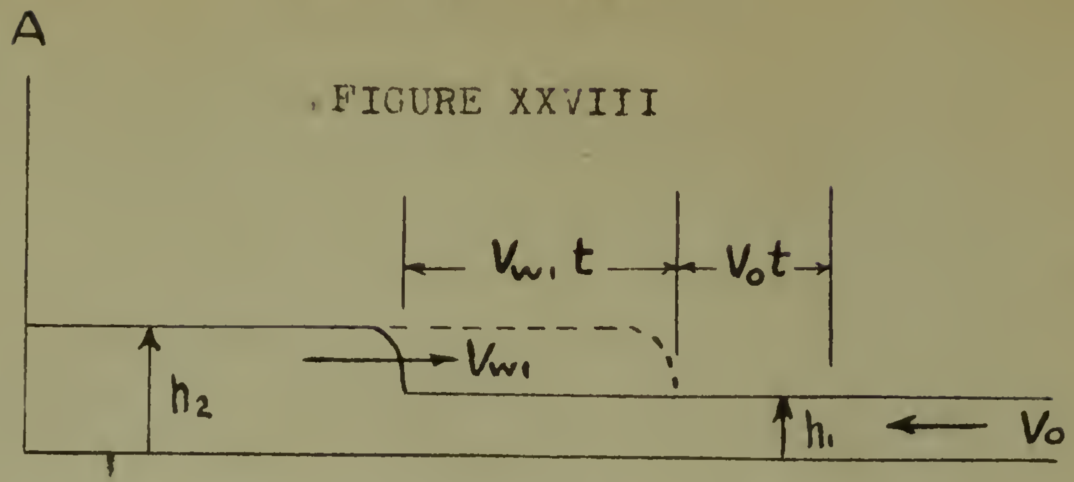
$$\frac{p_{02}}{p_{01}} = \left(\frac{H_2}{H_1}\right)^3 \tag{20}$$

With these equalities, the results plotted for a hydraulic jump can be applied to a compression wave in gas. For purposes of qualitative analysis, the present plots are adequate.

APPENDIX "B"

THEORETICAL ANALYSIS

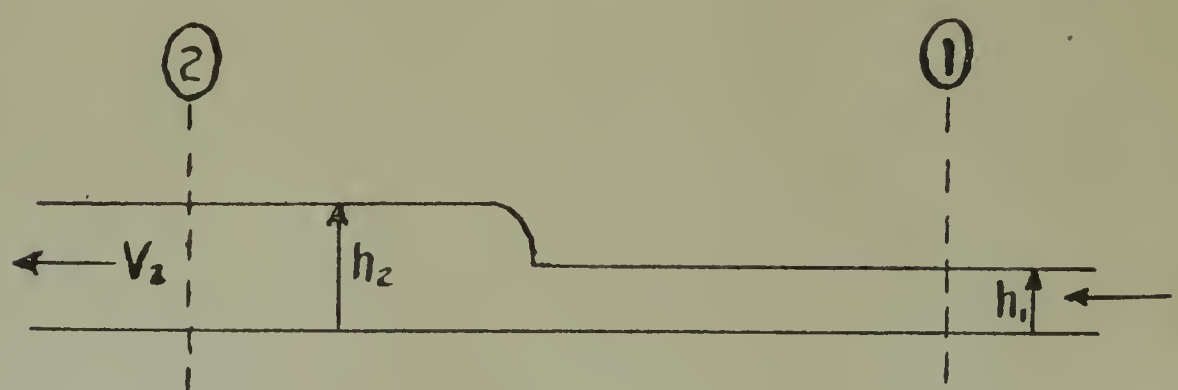
FIGURE XXVIII



In the experimental apparatus, water with velocity, V_o, and static head, h_1, flows through the channel. When the gate at A interrupts the flow, a hydraulic jump travels back toward the entrance with a velocity V_{w1}. Water behind this wave is at a height h_2 and velocity zero. Referring to Figure I, this is analogous to the first compression (steps 1 to 4) in the Comprex cycle.

Now consider the observer moving with a velocity V_{w1}. The wave becomes a stationary hydraulic jump. The velocity of the water approaching the jump is $V_1 = V_o + V_{w1}$. The velocity of the water following the jump is $V_2 = V_{w1}$.

FIGURE XXIX



For this steady state condition, the equation of continuity may be written:

$$\rho A_1 V_1 = \rho A_2 V_2 \tag{21}$$

Change in momentum per unit time is:

$$\frac{\Delta(MV)}{t} = \frac{\rho A_2 V_2^2}{g} - \frac{\rho A_1 V_1^2}{g} \tag{22}$$

Total pressure at (1) $= P_1 = \rho A_1 Z_1$ (23)

Total pressure at (2) $= P_2 = \rho A_2 Z_2$ (24)

Hence:

$$\frac{A_2 V_2^2 - A_1 V_1^2}{g} = A_1 Z_1 - A_2 Z_2 \tag{25}$$

(neglecting friction at boundaries)

Collecting terms:

$$A_2\left[\frac{V_2^2}{g} + Z_2\right] = A_1\left[\frac{V_1^2}{g} + Z_1\right] \tag{26}$$

For a triangular channel:

$$A = C h^2 \tag{27}$$

$$z = \frac{h}{3} \tag{28}$$

Then:

$$\frac{V_2}{V_1} = \frac{A_2}{A_1} = \frac{1}{\left(\frac{h_2}{h_1}\right)^2} \tag{29}$$

Substituting in (29)

$$C h_2^2 \left(\frac{V_2^2}{g}\right) + \frac{C h_2^3}{3} = C h_1^2 \left(\frac{V_1^2}{g}\right) + \frac{C h_1^3}{3} \tag{30}$$

Dividing by $C h_2 V_1^2$:

$$\frac{1}{g}\left(\frac{V_2}{V_1}\right)^2 + \frac{h_2}{3V_1^2} = \frac{1}{g}\left(\frac{h_1}{h_2}\right)^2 + \left(\frac{h_1}{h_2}\right)^2 \frac{h_1}{3V_1^2} \tag{31}$$

Substituting $\left(\frac{h_1}{h_2}\right)^2$ for $\frac{V_2}{V_1}$, rearranging:

$$M_1 = \frac{V_1}{\sqrt{\frac{g h_1}{2}}} = \left(\frac{h_2}{h_1}\right) \sqrt{\frac{2}{3} \frac{\left[\left(\frac{h_2}{h_1}\right)^3 - 1\right]}{\left[\left(\frac{h_2}{h_1}\right)^2 - 1\right]}} \tag{32}$$

$$\frac{M_1}{M_2} = \frac{V_1}{V_2}\sqrt{\frac{h_2}{h_1}} = \left(\frac{h_2}{h_1}\right)^{5/2} \tag{33}$$

$$M_2 = \frac{1}{\left(\frac{h_2}{h_1}\right)^{3/2}} \sqrt{\frac{2}{3} \frac{\left[\left(\frac{h_2}{h_1}\right)^3 - 1\right]}{\left[\left(\frac{h_2}{h_1}\right)^2 - 1\right]}} \tag{34}$$

$$M_0 = \frac{V_0}{\sqrt{\frac{g h_1}{2}}} \tag{35}$$

From Figure XXVIII and continuity:

$$\rho A_1 V_0 t = \rho (A_2 - A_1) V_{w1} t \tag{36}$$

but $V_{w1} = V_1$ (37)

hence:

$$V_0 = \left(\frac{A_2 - A_1}{A_1}\right) V_2 \qquad (38)$$

Substituting in (35):

$$M_0 = M_1 \left[\frac{A_2 - A_1}{A_1}\right] \frac{V_2}{V_1} = M_1 \left[1 - \left(\frac{h_1}{h_2}\right)^2\right] \qquad (39)$$

$$M_0 = \frac{1}{\left(\frac{h_2}{h_1}\right)} \sqrt{\frac{2}{3}\left[\left(\frac{h_2}{h_1}\right)^3 - 1\right]\left[\left(\frac{h_2}{h_1}\right)^2 - 1\right]} \qquad (40)$$

Let us now define the total head of the water.

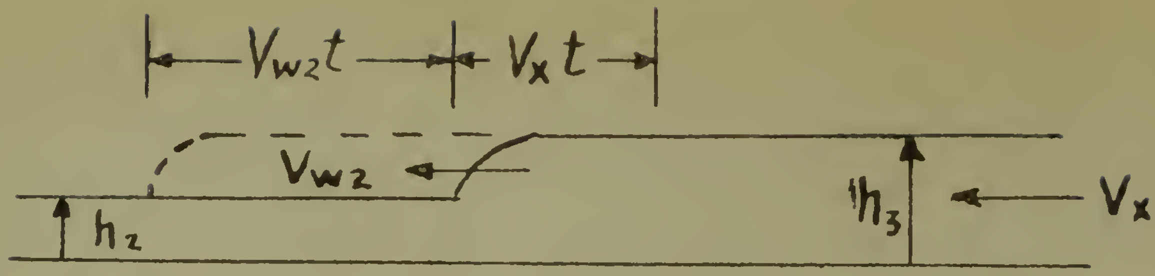
$$H = \frac{V^2}{2g} + h = h\left[\frac{M^2}{4} + 1\right] \qquad (41)$$

We may consider $\frac{H_2}{H_1}$ a measure of the efficiency of the hydraulic jump.

$$\frac{H_2}{H_1} = \frac{h_2}{h_1} \frac{\left[\frac{M_2^2}{4} + 1\right]}{\left[\frac{M_1^2}{4} + 1\right]} \qquad (42)$$

Again referring to Figure I, the second compression (steps 5 through 7) can be seen to be analogous to a second hydraulic jump.

FIGURE XXX



Water with zero velocity is in the channel. The gate at the inlet end is opened and water at a higher head than that in the channel enters with a velocity V_x. A hydraulic jump is propagated down the channel with velocity = V_{w2}. Water behind the wave has a head h_3 and a velocity = V_x.

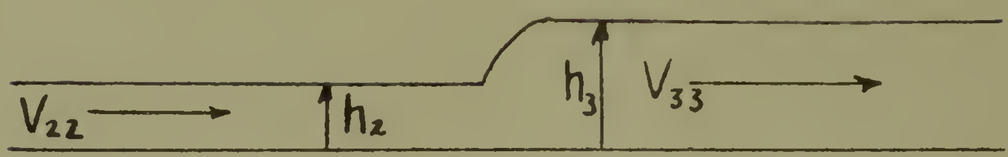
Now consider the observer moving with the velocity V_{w2}. The picture becomes exactly similar to the one for for the first hydraulic jump. The wave is stationary. Water approaches the wave with a velocity $V_{22} = V_{w2}$ and leaves with a velocity $V_{33} = V_{w2} - V_x$.

Therefore we can write at once:

$$M_{22} = \frac{h_3}{h_2}\sqrt{\frac{2}{3}\frac{\left[\left(\frac{h_3}{h_2}\right)^3 - 1\right]}{\left[\left(\frac{h_3}{h_2}\right)^2 - 1\right]}} \qquad (43)$$

$$M_{33} = \frac{1}{\left(\frac{h_3}{h_2}\right)^{3/2}}\sqrt{\frac{2}{3}\frac{\left[\left(\frac{h_3}{h_2}\right)^3 - 1\right]}{\left[\left(\frac{h_3}{h_2}\right)^2 - 1\right]}} \qquad (44)$$

FIGURE XXXI



From Figure XXX and continuity:

$$\rho V_x t A_3 = \rho V_{w2} t \, (A_3 - A_2) \qquad (45)$$

$$V_{w2} = V_{33} + V_x \qquad (46)$$

Substituting and rearranging:

$$V_x = V_{33}\left[\frac{A_3}{A_2} - 1\right] = V_{33}\left[\left(\frac{h_3}{h_2}\right)^2 - 1\right] \qquad (47)$$

Dividing both sides by $\sqrt{\dfrac{g h_3}{2}}$:

$$M_x = \frac{V_x}{\sqrt{\frac{g h_3}{2}}} = M_{33}\left[\left(\frac{h_3}{h_2}\right)^2 - 1\right] \qquad (48)$$

$$M_x = \frac{1}{\left(\frac{h_3}{h_2}\right)^{3/2}} \sqrt{\frac{2}{3}\left[\left(\frac{h_3}{h_2}\right)^3 - 1\right]\left[\left(\frac{h_3}{h_2}\right)^2 - 1\right]} \qquad (49)$$

From symmetry:

$$\frac{V_{33}}{V_{22}} = \frac{1}{\left(\frac{h_3}{h_2}\right)^2} \qquad (50)$$

$$\frac{H_{33}}{H_{22}} = \frac{h_3\left[\frac{M_{33}^2}{4}+1\right]}{h_2\left[\frac{M_{22}^2}{4}+1\right]} \tag{51}$$

Thus it is seen that, from the point of view of an observer moving with the velocity of the jump, curves plotted for the first hydraulic jump for values of $\left(\frac{h_2}{h_1}\right)$ are valid for the second hydraulic jump and values of $\left(\frac{h_3}{h_2}\right)$

If we consider the ratios $\frac{H_2}{H_1} = \frac{h_2}{h_1}\frac{\left[\frac{M_2^2}{4}+1\right]}{\left[\frac{M_1^2}{4}+1\right]}$ and $\frac{H_{33}}{H_{22}} = \frac{h_3}{h_2}\frac{\left[\frac{M_{33}^2}{4}+1\right]}{\left[\frac{M_{22}^2}{4}+1\right]}$ as measures of the efficiencies of the first and second hydraulic jumps, their product $\left(\frac{H_2}{H_1}\right)\left(\frac{H_{33}}{H_{22}}\right)$ may be considered a measure of the efficiency of the two jumps in series.

Now with the overall static head ratio, $\frac{h_3}{h_1}$, fixed, let us determine what percentage of this head ratio should be built up in the first stage. We wish to find the maximum value of $\left(\frac{H_2}{H_1}\right)\left(\frac{H_{33}}{H_{22}}\right)$ for varying values of $\frac{h_2}{h_1}$ while holding $\left(\frac{h_3}{h_1}\right)$ constant.

Suppose we select $\frac{h_2}{h_1} = \frac{h_3}{h_2}$ as a starting point and investigate what happens as $\frac{h_2}{h_1}$ is varied in either direction.

For this condition, since $\frac{h_3}{h_1}$ is constant,

$\frac{h_2}{h_1} = \frac{h_3}{h_2} = \sqrt{\frac{h_3}{h_1}} = X_0$. Now if $\frac{h_2}{h_1}$ varies, say $\frac{h_2}{h_1} = KX_0$, then $\frac{h_3}{h_2} = \frac{X_0}{K}$. Now $\frac{H_2}{H_1}$ is a function of $\frac{h_2}{h_1}$, and $\frac{H_{33}}{H_{22}}$ is the same function of $\frac{h_3}{h_2}$.

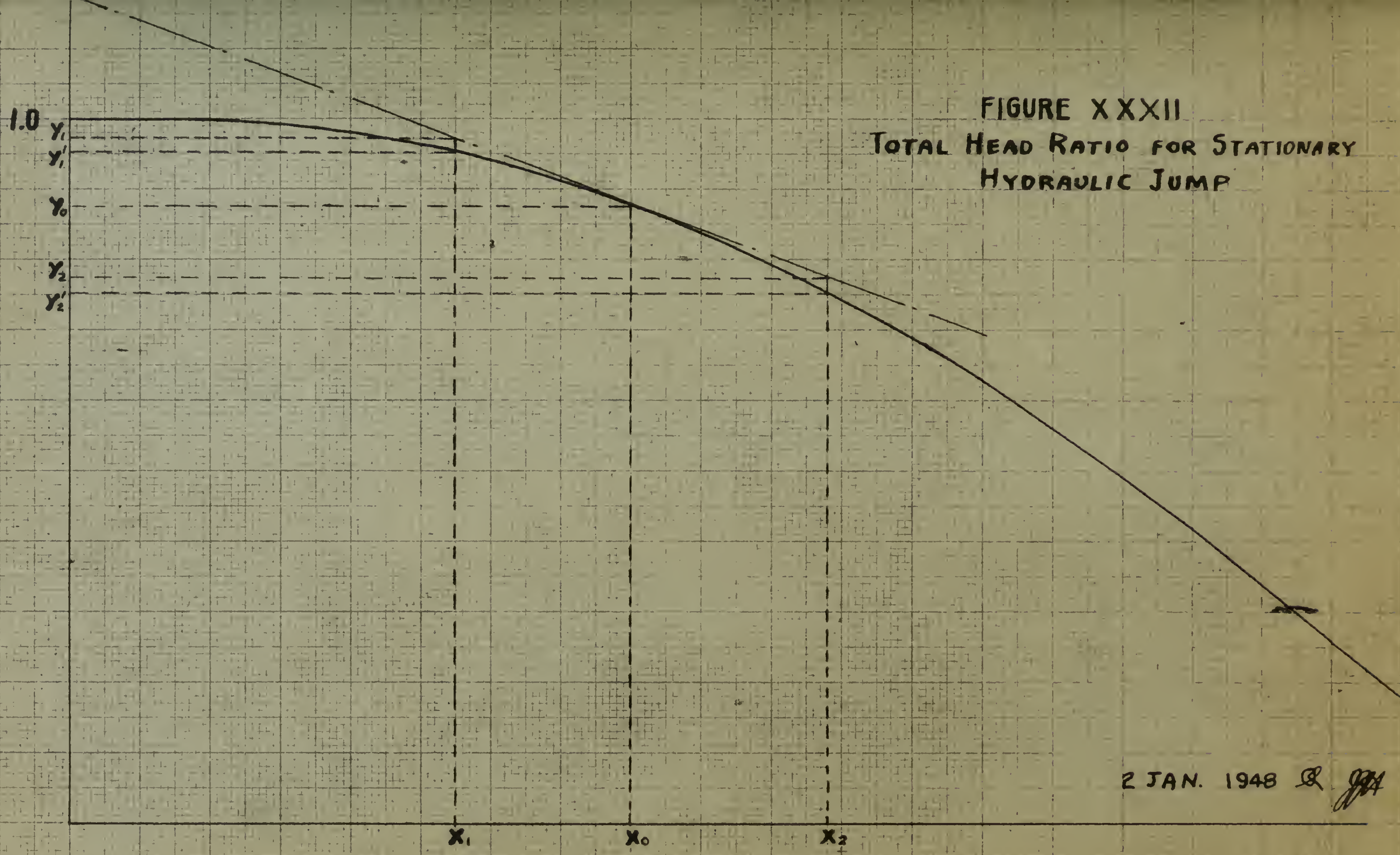


FIGURE XXXII
TOTAL HEAD RATIO FOR STATIONARY
HYDRAULIC JUMP

This function is plotted in Figure XXXII.

Referring to Figure XXXII:

When $\frac{h_2}{h_1} = \frac{h_3}{h_2}$, the corresponding values of $\frac{H_2}{H_1}$ and $\frac{H_{33}}{H_{22}}$ are equal and their product is Y_0^2. If $\frac{h_2}{h_1}$ becomes KX_0, $\frac{h_3}{h_2}$ becomes $\frac{X_0}{K}$ and the corresponding product is $Y_1'Y_2'$. If the function were a straight line, Y_1' would become Y_1 and Y_2' would become Y_2. The equation for any straight line is $y = mx + b$, where m is the slope and b is the intercept on the y-axis. From the figure, we have:

$$Y_0 = mX_0 + b$$

$$Y_1 = KmX_0 + b$$

$$Y_2 = \frac{mX_0}{K} + b$$

$$Y_0^2 - Y_1Y_2 = bmX_0\left[2 - \left(K + \frac{1}{K}\right)\right]$$

In this case, b is always positive and m is always negative, hence Y_0^2 is always greater than Y_1Y_2 since K cannot be negative. Since it is obvious that $Y_1Y_2 > Y_1'Y_2'$, we may conclude that the optimum condition occurs when $\frac{h_2}{h_1} = \frac{h_3}{h_2} = \sqrt{\frac{h_3}{h_1}}$.

Figure VI is a plot of $\left(\frac{H_2}{H_1}\right)\left(\frac{H_{33}}{H_{22}}\right)$ versus $\frac{h_2/h_1}{h_3/h_2}$ for the two special cases where $\frac{h_3}{h_1} = 1.5$ and $\frac{h_3}{h_1} = 2.0$.

It is seen that the maximum occurs where $\frac{h_2}{h_1} = \frac{h_3}{h_1}$. Also both curves are very flat over a wide range of $\frac{h_2}{h_1}$, which indicates that moderate variations from the optimum

condition will have a negligible effect over a wide range of $\dfrac{h_3}{h_1}$.

APPENDIX "C"

Summary of Theoretical Calculations

M_0	h_2/h_1	M_1	M_2	V_2/V_1	H_2/H_1	h_2/H_0	$M_0^2/(h_2/h_1)$
0	1.00	1.00	1.00	1.00	1.00	1.00	0
0.099	1.05	1.063	.942	.907	.999	1.047	.009
0.195	1.10	1.127	.890	.827	.999	1.090	.035
0.290	1.15	1.193	.841	.757	.998	1.126	.073
0.386	1.20	1.250	.798	.694	.996	1.157	.124
0.478	1.25	1.329	.761	.640	.993	1.183	.183
0.570	1.30	1.398	.726	.592	.988	1.202	.250
0.664	1.35	1.468	.693	.548	.983	1.216	.326
0.753	1.40	1.540	.664	.511	.976	1.226	.405
0.836	1.45	1.614	.637	.476	.968	1.230	.493
0.939	1.50	1.689	.612	.449	.957	1.229	.587
1.030	1.55	1.764	.588		.947		
1.119	1.60	1.838	.568	.391	.937	1.219	.783
1.215	1.65	1.919	.546		.923		
1.307	1.70	1.999	.530	.346	.910	1.192	1.003
	1.75						
1.489	1.80	2.155	.495	.309	.884	1.158	1.232
1.590	1.85	2.250	.482		.862		
1.682	1.90	2.327	.467	.277	.851	1.112	1.492
1.779	1.95	2.410	.454	.262	.836		
1.870	2.00	2.492	.441	.250	.822	1.067	1.750
1.168	2.15	2.765	.408	.216	.768		
2.259	2.20	2.847		.207			
2.457	2.30	3.029		.189			
2.677	2.40	3.250		.176			
2.863	2.50	3.408		.160			
3.066	2.60	3.598		.148			
3.278	2.70	3.799		.137			

FOR $V_0 = V_x$

M_0	h_2/h_1	h_3/h_2	M_x	h_3/h_1	H_2/H_1	H_{33}/H_{22}	$\left(\frac{H_2}{H_1}\right)\left(\frac{H_{33}}{H_{22}}\right)$	$(h_2/h_1)/(h_3/h_2)$
0	1.0	1.0	0	1.0	1.0	1.0	1.0	1.0
0.097	1.048	1.047	0.096	1.097	0.999	0.999	0.999	1.001
0.206	1.108	1.100	0.186	1.218	0.999	0.999	0.999	1.007
0.310	1.165	1.150	0.271	1.340	0.998	0.998	0.997	1.014
0.430	1.227	1.200	0.352	1.471	0.995	0.996	0.990	1.022
0.536	1.285	1.250	0.427	1.607	0.990	0.993	0.982	1.028
0.666	1.354	1.300	0.501	1.761	0.983	0.988	0.971	1.041
0.789	1.420	1.350	0.572	1.917	0.973	0.983	0.957	1.052
0.918	1.490	1.400	0.636	2.085	0.960	0.976	0.926	1.063
1.067	1.570	1.450	0.710	2.275	0.943	0.968	0.912	1.083
1.197	1.640	1.500	0.766	2.460	0.926	0.958	0.886	1.093
1.500	1.803	1.600	0.883	2.890	0.881	0.937	0.827	1.128
1.814	1.970	1.700	1.003	3.350	0.830	0.910	0.753	1.158

FOR $\frac{h_2}{h_1} = \frac{h_3}{h_2}$

M_0	$h_2/h_1 = h_3/h_2$	h_3/h_1	M_x	$\frac{H_2}{H_1} = H_{33}/H_{22}$	$\left(\frac{H_2}{H_1}\right)\left(\frac{H_{33}}{H_{22}}\right) = \left(\frac{H_2}{H_1}\right)^2$
0	1.0	1.0	0	1.0	1.0
0.097	1.047	1.096	0.097	0.999	0.999
0.206	1.108	1.227	0.196	0.999	0.999
0.310	1.165	1.357	0.285	0.998	0.996
0.430	1.227	1.505	0.385	0.995	0.989
0.536	1.285	1.651	0.472	0.990	0.980
0.666	1.354	1.833	0.573	0.983	0.966
0.789	1.420	2.016	0.661	0.973	0.945
0.918	1.490	2.220	0.750	0.960	0.920
1.067	1.570	2.465	0.852	0.943	0.889
1.197	1.640	2.690	0.935	0.926	0.858
1.500	1.803	3.260	1.117	0.881	0.776
1.814	1.970	3.881	1.295	0.830	0.688

FOR $\frac{h_3}{h_1}$ = CONSTANT

$\left(\frac{h_2}{h_1}\right) / \left(\frac{h_3}{h_2}\right)$	$\frac{h_2}{h_1}$, $\frac{h_3}{h_1} = 2.0$	$\frac{h_2}{h_1}$, $\frac{h_3}{h_1} = 1.5$	$\left(\frac{H_2}{H_1}\right)\left(\frac{H_{33}}{H_{22}}\right)$, $\frac{h_3}{h_1} = 2.0$	$\left(\frac{H_2}{H_1}\right)\left(\frac{H_{33}}{H_{22}}\right)$, $\frac{h_3}{h_1} = 1.5$
0.5	1.00		0.819	
0.667	1.15	1.00	0.889	0.958
0.8	1.26	1.10	0.930	0.982
1.0	1.41	1.22	0.950	0.990
1.2	1.55	1.34	0.936	0.984
1.4	1.67	1.45	0.915	0.968
1.5	1.73	1.5	0.900	0.958
1.6	1.79		0.885	
1.8	1.90		0.851	
2.0	2.0		0.819	

Summary of Experimental Calculations

Run	M_o	h_2/h	V/V_1	h_2/H_o	H_2/H_1
1	0.453	1.22	63	1.16	1.00
2	0.614	1.35	54	1.23	1.02
3	0.592	1.31	.56	1.21	.99
4	0.683	1.38	.52	1.23	1.00
5	0.152	1.07	74	1.01	1.01
6	0.317	1.17	.73	1.14	1.01
7	0.459	1.23	.63	1.17	1.00
8	0.609	1.31	.57	1.20	.99
9	0.568	1.31	.58	1.21	1.00
10	0.392	1.21	.69	1.16	1.01
11	0.260	1.14	.77	1.12	1.01
12	0.134	1.06	.86	1.05	1.00
13	0.67	1.35	.53	1.21	.99
14	0.205	1.11	.81	1.09	1.00
15	0.191	1.10	.67	1.05	.91
16	0.154	1.07	.85	1.07	1.00
17	0.264	1.23	.76	1.11	1.00
18	0.271	1.13	.77	1.11	1.00
19	0.388	1.20	.67	1.15	1.00
20	0.40	1.21	.67	1.17	1.01
21	0.52	1.26	.61	1.18	1.00
22	0.53	1.27	.60	1.19	.99
23	0.67	1.35	.52	1.21	.99
24	0.65	1.33	.53	1.20	.99
25	0.84	1.45	.42	1.24	1.02
26	0.70	1.36	.51	1.21	.99
27	0.72	1.37	.51	1.21	.98
28	0.53	1.30	.56	1.19	.99
29	2.74	2.61	.14	.90	.75
30	2.75	2.58	.14	.80	.73
31	2.70	2.61	.16	.92	.75
32	2.71	2.55	.15	.90	.73
33	2.80	2.58	.15	.86	.71
34	1.97	2.10	.23	1.06	.83
35	2.32	2.35	.18	1.00	.81
36	2.16	2.18	.20	1.00	.79
37	1.89	2.04	.23	1.08	.85
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.85

APPENDIX "D"

SAMPLE CALCULATIONS

FOR THE THEORETICAL ANALYSIS :

1 LET $\frac{h_2}{h_1} = 1.5$

$$\frac{V_2}{V_1} = \frac{1}{\left(\frac{h_2}{h_1}\right)^2} = \underline{\underline{0.444}}$$

$$M_1 = \frac{h_2}{h_1}\sqrt{\frac{2}{3}\frac{\left[\left(\frac{h_2}{h_1}\right)^3 - 1\right]}{\left[\left(\frac{h_2}{h_1}\right)^2 - 1\right]}} = \underline{\underline{1.689}}$$

$$M_2 = \frac{M_1}{\left(\frac{h_2}{h_1}\right)^{5/2}} = \underline{\underline{0.612}}$$

$$M_0 = M_1\left[1 - \frac{1}{\left(\frac{h_2}{h_1}\right)^2}\right] = \underline{\underline{0.939}}$$

$$\frac{H_2}{H_1} = \left(\frac{h_2}{h_1}\right)\frac{\left[\frac{M_2^2}{4} + 1\right]}{\left[\frac{M_1^2}{4} + 1\right]} = \underline{\underline{0.958}}$$

$$\frac{h_2}{H_0} = \frac{h_2/h_1}{\left[\frac{M_0^2}{4} + 1\right]} = \underline{\underline{1.229}}$$

FOR $\frac{h_3}{h_2} = \frac{h_2}{h_1} = 1.5$:

$$\frac{h_3}{h_1} = \left(\frac{h_3}{h_2}\right)\left(\frac{h_2}{h_1}\right) = \underline{\underline{2.25}}$$

$$\left(\frac{H_{33}}{H_{22}}\right)\left(\frac{H_2}{H_1}\right) = \left(\frac{H_2}{H_1}\right)^2 = \underline{\underline{0.918}}$$

FOR $\frac{h_3}{h_1}$ = CONSTANT = 2.0 :

LET $\frac{h_2/h_1}{h_3/h_2} = 0.8 = \frac{\left(\frac{h_2}{h_1}\right)^2}{\frac{h_3}{h_1}}$

$$\frac{h_2}{h_1} = \sqrt{(0.8)(2.0)} = \underline{\underline{1.26}}$$

$$\frac{h_3}{h_2} = \frac{\left(\frac{h_2}{h_1}\right)}{0.8} = \underline{\underline{1.58}}$$

From the curve of $\frac{H_2}{H_1}$ vs. M_0 :

FOR $\frac{h_2}{h_1} = 1.26$, $\frac{H_2}{H_1} = \underline{\underline{0.992}}$

FOR $\frac{h_3}{h_2} = 1.58$, $\frac{H_{33}}{H_{22}} = \underline{\underline{0.939}}$

$$\left(\frac{H_2}{H_1}\right)\left(\frac{H_{33}}{H_{22}}\right) = (0.992)(0.939) = \underline{\underline{0.930}}$$

The curve of $\frac{h_3}{h_1}$ for equal stream velocities was constructed graphically in order to minimize tabulation processes. We may write:

$$M_0^2 = \frac{V_0^2}{\left[\frac{g h_1}{2}\right]}$$

$$M_x^2 = \frac{V_x^2}{\left[\frac{g h_3}{2}\right]}$$

Dividing:

$$\frac{M_0^2}{M_x^2} = \frac{V_0^2}{V_x^2}\left(\frac{h_3}{h_1}\right) = \frac{V_0^2}{V_x^2}\left(\frac{h_3}{h_2}\right)\left(\frac{h_2}{h_1}\right)$$

Now if $V_0 = V_x$, we have :

$$\frac{M_0^2}{\left(\frac{h_2}{h_1}\right)} = M_x^2\left(\frac{h_3}{h_2}\right)$$

By comparison of equations (40) and (49), it is seen that if we substitute $\frac{h_3}{h_2}$ for $\frac{h_2}{h_1}$ in equation (40) the resulting value of M_o^2 is equivalent to $M_x^2\left(\frac{h_3}{h_2}\right)$. This suggests a graphical method of constructing $\frac{h_3}{h_1}$ for $V_o = V_x$. Refer to Figure XXXIII. For any value of $\frac{h_3}{h_2}$ we may enter the plot of $\frac{h_2}{h_1}$ vs. M_o and pick off the numerical value of $M_x^2\left(\frac{h_3}{h_2}\right)$, since the value of M_o^2 corresponding to this $\frac{h_3}{h_2}$ is equal to $M_x^2\left(\frac{h_3}{h_2}\right)$. A curve of $\frac{M_o^2}{h_2/h_1}$ vs. M_o has been plotted on Figure XXXIII. We enter the vertical scale with the value of $M_x^2\left(\frac{h_3}{h_2}\right)$. The corresponding point on the curve of $\frac{M_o^2}{h_2/h_1}$ vs. M_o gives us the value of M_o and $\frac{h_2}{h_1}$ for the selected value of $\frac{h_3}{h_2}$. The product $\left(\frac{h_3}{h_2}\right)\left(\frac{h_2}{h_1}\right)$ yields $\frac{h_3}{h_1}$. This is plotted vs. M_o. In this way $\frac{h_3}{h_1}$ vs. M_o for equal stream velocities was plotted. The curve of M_x vs. M_o for equal stream velocities was plotted by dividing the values of $M_x\sqrt{\frac{h_3}{h_2}}$ obtained above by $\sqrt{\frac{h_3}{h_2}}$ and plotting the resulting M_x vs. M_o. The curve of M_x vs. M_o for $\frac{h_3}{h_2} = \frac{h_2}{h_1}$ was obtained by plotting the values of M_x obtained above vs. an M_o corresponding to the value $M_x\sqrt{\frac{h_3}{h_2}}$.

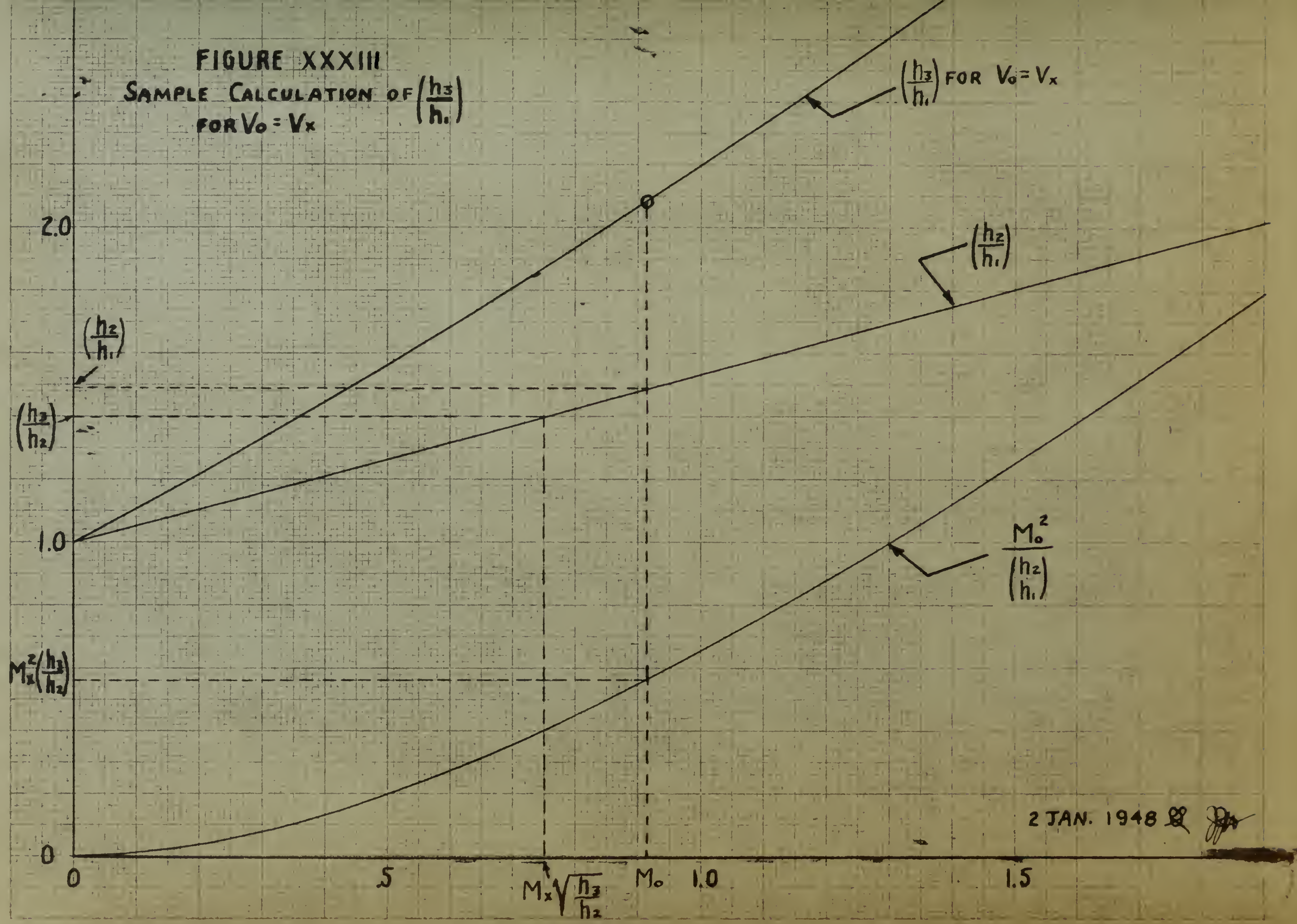


FIGURE XXXIII
SAMPLE CALCULATION OF $\left(\frac{h_3}{h_1}\right)$
FOR $V_0 = V_x$
$\left(\frac{h_3}{h_1}\right)$ FOR $V_0 = V_x$
$\left(\frac{h_2}{h_1}\right)$
$\frac{M_0^2}{\left(\frac{h_2}{h_1}\right)}$
2.0
1.0
0
$\left(\frac{h_2}{h_1}\right)$
$\left(\frac{h_3}{h_2}\right)$
$M_x^2\left(\frac{h_3}{h_2}\right)$
0
.5
1.0
1.5
$M_x\sqrt{\frac{h_3}{h_2}}$
M_0
2 JAN. 1948

FOR THE EXPERIMENTAL POINTS:

RUN No. 20

$Q = \left[\frac{W}{t}\right] C_2$, WHERE C_2 IS STREAM IMPACT CORRECTION (SEE PAGE 16).

AVERAGE VALUE OF $C_2 = 0.995$

$$Q = \frac{(200)(0.995)}{26.75} = \underline{\underline{7.44 \text{ lb./sec.}}}$$

$h_1 = 13.36$ cm. (MEASURED)

$h_2 = 16.16$ cm. (MEASURED)

$$\frac{h_2}{h_1} = \frac{16.16}{13.36} = \underline{\underline{1.210}}$$

$$\rho = (62.4)(.98) = 61.2 \text{ lb/ft.}^3 = \underline{\underline{2.16 \times 10^{-3} \text{ lb/cm}^3}}$$

$$A_1 = C h_1^2 = \tan\left(\frac{62.5}{2}\right)^\circ h_1^2 = 0.608\, h_1^2$$

$$V_0 = \frac{Q}{\rho A_1} = \frac{Q}{h_1^2}\left(\frac{10^3}{[.608][2.16]}\right) = 761 \frac{Q}{h_1^2}$$

$$V_0 = \frac{(761)(7.44)}{(13.36)^2} = \underline{\underline{32.2 \text{ cm/sec.}}}$$

$$M_0 = \frac{V_0}{\sqrt{\frac{g h_1}{2}}} = \frac{32.2}{\sqrt{\frac{(980)(13.36)}{2}}} = \underline{\underline{0.398}}$$

$$\frac{h_2}{H_0} = \frac{h_2}{\frac{V_0^2}{2g} + h_1} = \frac{h_2/h_1}{\frac{M_0^2}{4} + 1} = \frac{1.210}{\frac{(.398)^2}{4} + 1} = \underline{\underline{1.165}}$$

$$V_w = \frac{\text{DISTANCE WAVE TRAVELS}}{\text{TIME}} = \frac{248.7 \text{ cm.}}{3.8 \text{ sec.}} = \underline{\underline{65.4 \text{ cm/sec}}}$$

$$V_1 = V_0 + V_w = 32.2 + 65.4 = \underline{\underline{97.6 \text{ cm/sec.}}}$$

$$\frac{V_2}{V_1} = \frac{V_w}{V_1} = \frac{65.4}{97.6} = \underline{\underline{0.670}}$$

$$\frac{H_2}{H_1} = \frac{\frac{V_2^2}{2g} + h_2}{\frac{V_1^2}{2g} + h_1} = \frac{2.185 + 16.16}{4.86 + 13.36} = \underline{\underline{1.01}}$$

APPENDIX "E"

ORIGINAL DATA

RUN	DATE	FLOW IN LBS.	TIME OF FLOW (SEC.)	INITIAL STATIC HEAD	DYNAMIC HEAD	FINAL STATIC HEAD	WAVE TRAVEL IN CMS.	TIME OF TRAVEL
1	12/20/47	50	144.6	3.71 cm.		4.52 cm.	188.21	5.9 sec.
2		100	42.1	7.10		9.55	188.21	4.5
3		300	80.6	8.60		11.30	248.66	4.95
4		150	18.0	11.22		15.38		4.5
5		200	64.0	13.80		14.78		3.9
6		150	24.1	13.51		15.84		3.6
7		150	19.5	12.80		15.80		4.0
8		150	18.4	11.76		15.38		4.2
9		200	29.1	11.20		14.63		4.2
10		200	37.5	11.76		14.18		4.0
11		200	44.4	12.92		14.70		3.55
12		100	37.3	13.73		14.52		3.77
13		150	17.6	11.43		15.4		4.5
14	12/22/47	100	31.8	16.23		17.95		3.15
15		100	35.9	15.90		17.34		3.30
16		100	40.25	16.60		17.78		3.25
17		200	33.75	14.34		16.13		3.62
18		200	31.75	14.55		16.49		3.35
19		200	28.35	13.15		15.76		3.9
20		200	26.75	13.36		16.16		3.80
21		200	28.5	11.74		14.82		4.15
22		200	25.57	12.12		15.39		4.15
23		150	18.2	11.27		15.20		4.50
24		150	18.35	11.34		15.12		4.48
25		150	79.65	5.72		8.30	188.21	5.95
26		150	17.18	11.36		15.41	248.66	4.60
27		150	17.05	11.25		15.41	248.66	4.45
28		200	51.0	8.78		11.38	248.66	5.05
29	12/30/47	200	59.0	4.48		11.68	188.21	9.35
30		200	29.4	5.93		15.28		2.60
31		200	46.4	4.98		12.98		7.70
32		200	46.0	4.98		12.68		7.80
33		200	24.0	6.38		16.38		6.90
34		150	17.9	7.38		15.48		5.40
35		200	25.8	6.68		15.68		6.60
36		140	15.85	7.23		15.74		5.75
37		100	12.4	7.38		15.08		5.50
38		150	18.0	7.63		15.03		5.35

APPENDIX "F"

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